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ABSTRACT

This teacher resource material was written from the perspective that it is possible to improve number fact learning in the primary grades, and in remedial situations beyond those years, in ways that: (1) will help children learn all facts; (2) will emphasize the process as well as the answer; and (3) by capitalizing upon natural extensions of number fact work, will contribute significantly to major goals of today's curriculum. A major thrust of the paper has been to present a framework which teachers can use as a springboard for improving their present basic fact program, no matter what school adopted mathematics textbook is used. The framework is used to present sample lessons in addition, subtraction, multiplication and division. In addition, the document addresses program guidelines, remediation, use of games, meeting individual needs and extensions of the program. Blackline masters are included. (PK)

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TEACHING BASIC FACTS

An Activity Based Program

A Teacher Resource

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TEACHING BASIC FACTS

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Toward a Sound Basic Fact Program: A Philosophy of Teaching

A child's ability to provide answers to basic number combinations accurately and quickly is a basic stepping stone to success in other aspects of the elementary school mathematics curriculum. Written and mental computation, computational estimation and elements of problem solving rely on this skill. Typically, mastery of the basic "number facts" is not easily and quickly attained. It is not unusual for children in and beyond third grade to finger count or guess to obtain answers to unknown facts.

The school mathematics curriculum today is fuller than ever with demands to provide quality teaching in key areas: problem solving and applications, mental computation and computational estimation; and the processing and interpretation of data, both graphically and with appropriate use of technology. Teachers are being challenged to develop more adequate understandings of mathematical concepts, and to build "number sense" in practical and problematic situations.

To help students keep pace with the demands of today's curriculum and allow time for its more challenging aspects, there is a need to be more *efficient* in the teaching and learning of basic facts. Two major questions confront teachers:

- What approaches to number fact learning will bring students to a usable degree of mastery faster and foster better retention?

- What model of fact learning also reinforces concept learning and the

development of number relationships; and has a natural extension to mental mathematics and problem solving?

Baroody (1985) has endorsed a model which emphasizes "thinking strategies" and suggests that this approach to number fact learning and teaching typically provides more effective results than a drill approach alone. Baroody's proposal aligns itself to the theoretical model proposed by Case (1977; 1982) which may be used to explain one prominent way in which children differ in their learning and retention of basic facts.

Case's information processing model is based on the concept that the capacity of a child's *working memory* is limited. Working memory is interpreted as the amount of space available in an individual's mind for holding new data while doing the necessary mental work to give a response.

Case's research suggests that the capacity of one's working memory is increased by "chunking"--that is, grouping or relating data for more efficient storage and retrieval. The result is that greater amounts of data can be placed in one's working memory at any given time, and transfer to long term memory is accommodated more readily.

"Chunking" is the basis of a strategy approach to teaching and learning basic facts. Research evidence since the 1930's (e.g. Brownell and Chazal, 1937; Thiele, 1938; Swenson, 1949; Thornton, 1978; Rathmell, 1978; Leutzinger, 1979; Cook & Dossey, 1982; Thornton & Toohey, 1985) are consistent with Baroody's suggestion to examine the effectiveness of this approach.

Many teachers are beginning to recognize how effective the "strategy"

model of teaching basic facts can be and, as evidenced by the demand for staff development related to the topic, the approach is becoming more popular. When we teach strategies for recall before drilling facts for speed, we teach children how to help themselves when they forget. When strategies are taught first, it also is more likely that subsequent drill will "speed up" efficient processes rather than cumbersome ones for obtaining answers to UNKNOWN facts. Some textbooks are beginning to mix elements of a "strategy" approach with traditional approaches based on drill and grouping facts by size of sum, factor or product.

The concern of this author goes beyond the idea of merely teaching isolated strategies to a focus on:

(1) how carefully each strategy is taught;

(2) how the various strategies are sequenced; and

(3) how number fact work is related to major goals in school mathematics and the rest of the mathematics curriculum.

The emerging objective is to go beyond the use of techniques, hints, or strategies which help children learn *certain groups of facts* to the development of a **sound basic fact program** which:

(1) Will help children learn *all facts* ;

(2) Will emphasize the *process* as well as the answer; and

(3) By capitilizing upon natural extensions of number fact work, will contribute significantly to major goals of today's curriculum.

This teacher resource has been written from the perspective that it is possible to

improve number fact learning in the primary grades, and in remedial situations beyond those years, in a way which relates to this objective. A major thrust has been to **present a framework** which teachers can use as a springboard for improving their present basic fact program, no matter what school adopted mathematics textbook is used. This framework involves five basic steps and is described below.

A BASIC FRAMEWORK FOR TEACHING BASIC FACTS: FIVE TYPES OF ACTIVITIES

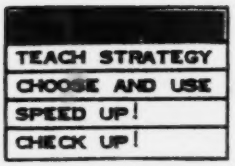
Goal: Fact memorization. Adequate concept understanding is assumed.

For all but very good memorizers. it is helpful to:

1. **LOOK AHEAD** to:
   - tomorrow
   - three weeks from now. Build prerequisites which make it easier/possible to use a strategy approach to teaching basic facts (p 16).
2. **TEACH STRATEGIES** which help children memorize *Unknown* facts. Action-based teaching is the key. Strategies which children find helpful are summarized on page 28. Group facts by strategy for recall, rather than size of sum, product or factor.
3. Help students **CHOOSE and USE** correct, efficient retrieval strategies for *Unknown* facts.
4. Provide short, daily drill activities to **SPEED UP** response to the point of automatic recall.

   In every classroom to about grade 4 and in remedial settings, daily drill on facts previously presented should occur. Not to do so would be the exception. The tendency is to "let addition go" while working on subtraction; to neglect both addition and subtraction when focusing on multiplication or division; to work only on division to the exclusion of facts for other operations, and so on. Rather, grade 1-2 teachers might plan a short addition drill near the beginning of each math period, and a short subtraction drill near the end. Teachers who also have introduced multiplication and division facts for memorization might alternate days for "jiffy drills": addition/subtraction one day, multiplication/division the next, regardless of the focus for the days' lesson.
5. **CHECK UP** on fact mastery through regular six- or eight-point fact quizzes (and occasional longer timed tests). Keep Cross Out lists (p. 7) current.

Getting Started



The following guidelines may prove useful as you look ahead to plan your basic fact program for the year.

Check Concept Understanding

A program stressing fact memorization naturally starts with a concept check for each operation involved. Given a written number fact, can a child use objects in a meaningful way to derive answers? Or, at a higher level, if the child "knows" the correct answer, can objects be used to show that an answer provided makes sense? A first basis for checking concept understanding is using objects in relation to what is written.

Children's understanding of a concept can grow as their experiences give them a broader basis for comparison and use. The final version of the booklet will expand on this idea and present other ways of assessing concept understanding prior to a focus on memorization of basic facts.

Select the Starting Point

The starting point for students studying a set of number facts for the first time will differ, of course, from that selected for older learners. The final version of this teacher resource will explore options and alternatives for "getting started" in a school year for a given operation for regular class instruction and for remedial programs.

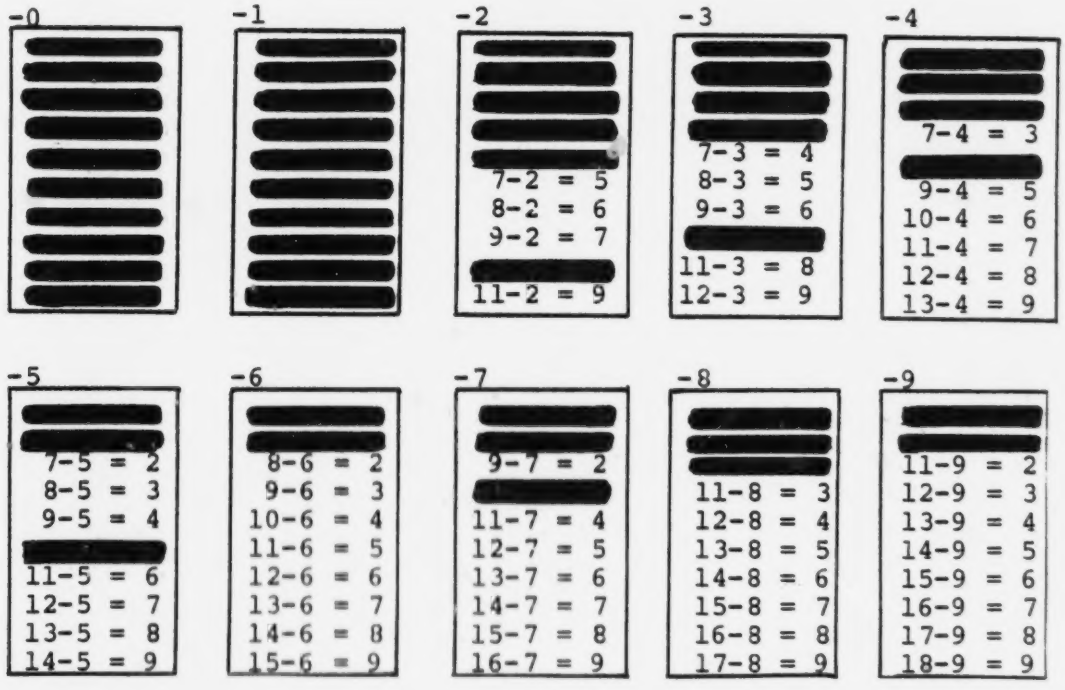
For older students, assessing what facts students know vs those they do not know plays an important role in "getting started." Carefully observing student response accuracy and rate in short drills, and analyzing errors made in computation helps teachers to identify, early in the year, which facts are known and which are difficult. In remedial situations, it is helpful if individual profiles can be obtained. In large classes, where it may not be manageable to test every student

In Remedial Settings, Test and Establish Individual Cross Out Sheets

The blackline masters for "Do the Easy Ones" provided on pages 9 through 12 or flash cards may be used for *individual* testing of students you feel lag furthest behind in fact mastery. This testing is written or oral. It is important to test students individually so you can keep track, during the test itself, of those facts which are answered correctly and quickly (within two seconds).

After the testing, the blackline masters of "Cross Out Sheets" on pages 13 and 14 can be used to **cross out** boldly, as in the sample below, **all those facts which students know well.** If children *count quickly* and can derive a correct answer within two seconds, cross out the fact. This two-second criterion for mastery typically can be met only when the fact is truly memorized or when short counts are involved. Long counts will involve a child beyond two seconds.

Subtraction Cross Out Sheet

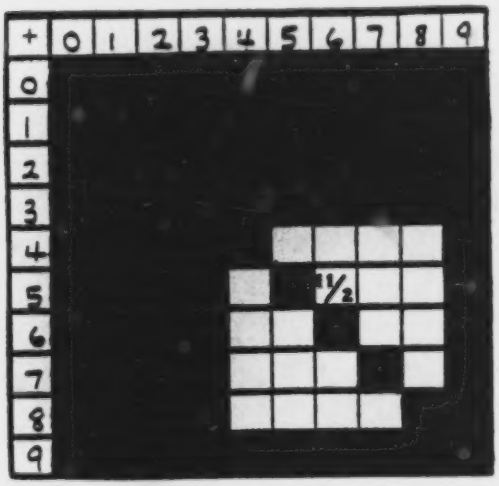


The Cross Out Sheets can be glued onto cardstock and used by students as bookmarks in their mathematics books. The suggestion is: 1) to use the Cross Out Sheets for diagnostic planning; and 2) to let students use them for seatwork and for all tests except standardized and basic fact tests until you have the chance to help students memorize *unknown* facts--those not yet crossed out. You know the students haven't memorized the facts. Can they otherwise do the work if you provide ready access to answers for unknown facts?

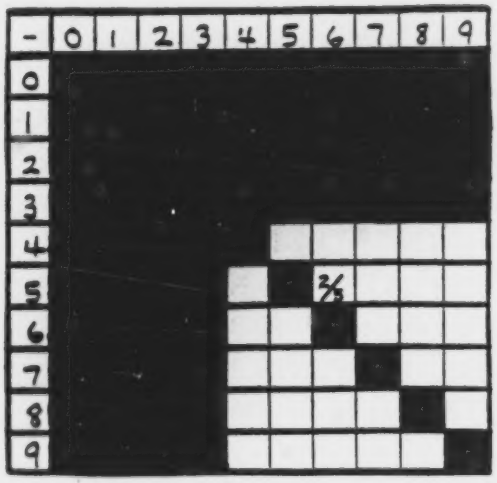
The Cross Out Sheets Should Get "Blacker".

Keep Cross Out Sheets current. Set weekly, individual goals for students. Select several facts for study which have a common strategy for recall. For example: "This week, use the pattern to learn 9s you don't know." Help students when you can during the week and, if possible, ask parents to help. Given that you've set reasonable goals, several facts should be learned and crossed out each week.

It typically is useful for teachers to keep a copy of the "Cross Out" status of their students. Teachers can keep duplicates of each student sheet or use recordkeeping forms like those on page 15 for this purpose. The samples below illustrate how to use these forms. Facts "known" at the time of initial testing are blackened. (A) indicates that "5+6" was learned and crossed out on November 2 (11/2). According to (B), the fact "11-5" was crossed out on February 5 (2/5).



(A)



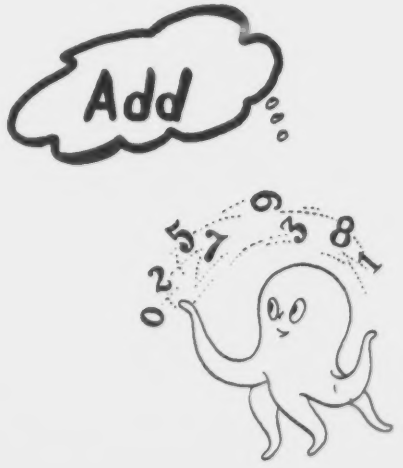
(B)

Do the Easy Ones: Addition

Quickly answer all those you know. It's all right to skip the hard ones.

3 + 3 = _____	7 + 6 = _____	8 + 8 = _____
8 + 1 = _____	2 + 5 = _____	3 + 4 = _____
7 + 7 = _____	2 + 2 = _____	8 + 9 = _____
4 + 8 = _____	4 + 4 = _____	4 + 7 = _____
3 + 9 = _____	4 + 9 = _____	3 + 5 = _____
2 + 3 = _____	4 + 3 = _____	4 + 0 = _____

2	4	6	5	9
+ 4	+ 5	+ 8	+ 6	+ 9
2	5	0	5	2
+ 7	+ 4	+ 6	+ 9	+ 6



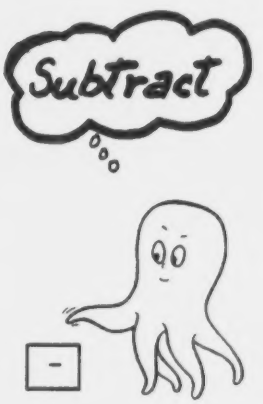
1 + 5 = _____	3 + 7 = _____	6 + 6 = _____
6 + 7 = _____	3 + 8 = _____	7 + 8 = _____
2 + 8 = _____	5 + 8 = _____	6 + 5 = _____
4 + 6 = _____	5 + 5 = _____	5 + 7 = _____
3 + 6 = _____	8 + 7 = _____	7 + 9 = _____

Do the Easy Ones: Subtraction

Quickly answer all those you know. It's all right to skip the hard ones.

14 - 7 = _____	13 - 6 = _____	8 - 4 = _____	7 - 2 = _____
6 - 4 = _____	9 - 8 = _____	16 - 9 = _____	9 - 0 = _____
10 - 7 = _____	10 - 6 = _____	8 - 1 = _____	10 - 3 = _____
14 - 8 = _____	11 - 4 = _____	17 - 8 = _____	14 - 5 = _____

17 -9	6 -1	12 -4	15 -7	8 -3	11 -7	14 -9	10 -4
18 -9	7 -0	14 -6	11 -6	13 -8	8 -5		
9 -6	11 -5	15 -8	10 -8	6 -6	11 -3		
9 -7	16 -8	9 -4	12 -7	9 -3	16 -7		



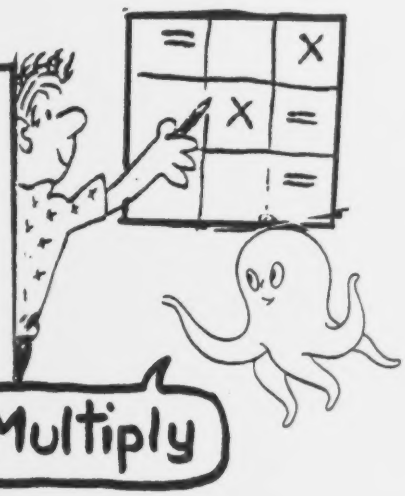
8 - 6 = _____	12 - 6 = _____	12 - 5 = _____	13 - 5 = _____
10 - 5 = _____	11 - 8 = _____	8 - 8 = _____	9 - 5 = _____
7 - 3 = _____	13 - 7 = _____	12 - 8 = _____	13 - 9 = _____

Do the Easy Ones: Multiplication

Quickly answer all those you know. It's all right to skip the hard ones.

3 x 3 = _____	8 x 1 = _____	2 x 3 = _____
4 x 4 = _____	3 x 5 = _____	4 x 3 = _____
0 x 4 = _____	8 x 9 = _____	3 x 4 = _____
4 x 9 = _____	3 x 9 = _____	7 x 7 = _____
4 x 8 = _____	8 x 8 = _____	4 x 7 = _____
2 x 2 = _____	7 x 6 = _____	2 x 5 = _____

0	5	8	5	2
x 6	x 9	x 6	x 6	x 7
4	2	5	6	9
x 5	x 6	x 4	x 8	x 9



2 x 8 = _____	3 x 8 = _____	5 x 5 = _____
8 x 7 = _____	6 x 7 = _____	7 x 9 = _____
5 x 7 = _____	5 x 8 = _____	3 x 6 = _____
4 x 6 = _____	1 x 5 = _____	6 x 6 = _____
3 x 7 = _____	6 x 6 = _____	7 x 8 = _____

Do the Easy Ones: Division

Quickly answer all those you know. It's all right to skip the hard ones.

9 ÷ 3 = _____	4 ÷ 2 = _____	42 ÷ 6 = _____	40 ÷ 5 = _____
6 ÷ 3 = _____	24 ÷ 3 = _____	8 ÷ 1 = _____	27 ÷ 3 = _____
16 ÷ 4 = _____	0 ÷ 7 = _____	45 ÷ 5 = _____	10 ÷ 2 = _____
42 ÷ 7 = _____	4 ÷ 4 = _____	35 ÷ 7 = _____	36 ÷ 9 = _____
		48 ÷ 6 = _____	56 ÷ 8 = _____

$8\overline{)40}$	$3\overline{)21}$	$7\overline{)14}$	$1\overline{)0}$	$6\overline{)54}$	$4\overline{)24}$	$6\overline{)30}$	$5\overline{)25}$
$4\overline{)20}$	$9\overline{)81}$	$9\overline{)63}$	$7\overline{)49}$	$1\overline{)2}$	$4\overline{)12}$		
$4\overline{)8}$	$3\overline{)12}$	$9\overline{)81}$	$3\overline{)15}$	$6\overline{)6}$	$6\overline{)12}$		
$4\overline{)32}$	$9\overline{)54}$	$3\overline{)0}$	$3\overline{)18}$	$7\overline{)21}$	$1\overline{)1}$		

56 ÷ 7 = _____	28 ÷ 4 = _____	18 ÷ 6 = _____
32 ÷ 8 = _____	16 ÷ 8 = _____	72 ÷ 8 = _____
35 ÷ 5 = _____	64 ÷ 8 = _____	36 ÷ 6 = _____
18 ÷ 9 = _____	27 ÷ 9 = _____	28 ÷ 7 = _____

Addition Cross Out Sheet

+0	+1	+2	+3	+4
0 +0 =0	0 +1 =1	0 +2 =2	0 +3 =3	0 +4 =4
1 +0 =1	1 +1 =2	1 +2 =3	1 +3 =4	1 +4 =5
2 +0 =2	2 +1 =3	2 +2 =4	2 +3 =5	2 +4 =6
3 +0 =3	3 +1 =4	3 +2 =5	3 +3 =6	3 +4 =7
4 +0 =4	4 +1 =5	4 +2 =6	4 +3 =7	4 +4 =8
5 +0 =5	5 +1 =6	5 +2 =7	5 +3 =8	5 +4 =9
6 +0 =6	6 +1 =7	6 +2 =8	6 +3 =9	6 +4 =10
7 +0 =7	7 +1 =8	7 +2 =9	7 +3 =10	7 +4 =11
8 +0 =8	8 +1 =9	8 +2 =10	8 +3 =11	8 +4 =12
9 +0 =9	9 +1 =10	9 +2 =11	9 +3 =12	9 +4 =13

+5	+6	+7	+8	+9
0 +5 =5	0 +6 =6	0 +7 =7	0 +8 =8	0 +9 =9
1 +5 =6	1 +6 =7	1 +7 =8	1 +8 =9	1 +9 =10
2 +5 =7	2 +6 =8	2 +7 =9	2 +8 =10	2 +9 =11
3 +5 =8	3 +6 =9	3 +7 =10	3 +8 =11	3 +9 =12
4 +5 =9	4 +6 =10	4 +7 =11	4 +8 =12	4 +9 =13
5 +5 =10	5 +6 =11	5 +7 =12	5 +8 =13	5 +9 =14
6 +5 =11	6 +6 =12	6 +7 =13	6 +8 =14	6 +9 =15
7 +5 =12	7 +6 =13	7 +7 =14	7 +8 =15	7 +9 =16
8 +5 =13	8 +6 =14	8 +7 =15	8 +8 =16	8 +9 =17
9 +5 =14	9 +6 =15	9 +7 =16	9 +8 =17	9 +9 =18

↑ GLUE TO 4X6 INDEX CARD ↑

Subtraction Cross Out Sheet

-0	-1	-2	-3	-4
0 -0 =0	1 -1 =0	2 -2 =0	3 -3 =0	4 -4 =0
1 -0 =1	2 -1 =1	3 -2 =1	4 -3 =1	5 -4 =1
2 -0 =2	3 -1 =2	4 -2 =2	5 -3 =2	6 -4 =2
3 -0 =3	4 -1 =3	5 -2 =3	6 -3 =3	7 -4 =3
4 -0 =4	5 -1 =4	6 -2 =4	7 -3 =4	8 -4 =4
5 -0 =5	6 -1 =5	7 -2 =5	8 -3 =5	9 -4 =5
6 -0 =6	7 -1 =6	8 -2 =6	9 -3 =6	10 -4 =6
7 -0 =7	8 -1 =7	9 -2 =7	10 -3 =7	11 -4 =7
8 -0 =8	9 -1 =8	10 -2 =8	11 -3 =8	12 -4 =8
9 -0 =9	10 -1 =9	11 -2 =9	12 -3 =9	13 -4 =9

-5	-6	-7	-8	-9
5 -5 =0	6 -6 =0	7 -7 =0	8 -8 =0	9 -9 =0
6 -5 =1	7 -6 =1	8 -7 =1	9 -8 =1	10 -9 =1
7 -5 =2	8 -6 =2	9 -7 =2	10 -8 =2	11 -9 =2
8 -5 =3	9 -6 =3	10 -7 =3	11 -8 =3	12 -9 =3
9 -5 =4	10 -6 =4	11 -7 =4	12 -8 =4	13 -9 =4
10 -5 =5	11 -6 =5	12 -7 =5	13 -8 =5	14 -9 =5
11 -5 =6	12 -6 =6	13 -7 =6	14 -8 =6	15 -9 =6
12 -5 =7	13 -6 =7	14 -7 =7	15 -8 =7	16 -9 =7
13 -5 =8	14 -6 =8	15 -7 =8	16 -8 =8	17 -9 =8
14 -5 =9	15 -6 =9	16 -7 =9	17 -8 =9	18 -9 =9

↑ GLUE TO 4X6 INDEX CARD ↑

13

Multiplication Cross Out Sheet

x0

0	x0 = 0
1	x0 = 0
2	x0 = 0
3	x0 = 0
4	x0 = 0
5	x0 = 0
6	x0 = 0
7	x0 = 0
8	x0 = 0
9	x0 = 0

x1

0	x 1 = 0
1	x 1 = 1
2	x 1 = 2
3	x 1 = 3
4	x 1 = 4
5	x 1 = 5
6	x 1 = 6
7	x 1 = 7
8	x 1 = 8
9	x 1 = 9

x2

0	x2 = 0
1	x2 = 2
2	x2 = 4
3	x2 = 6
4	x2 = 8
5	x2 = 10
6	x2 = 12
7	x2 = 14
8	x2 = 16
9	x2 = 18

x3

0	x3 = 0
1	x3 = 3
2	x3 = 6
3	x3 = 9
4	x3 = 12
5	x3 = 15
6	x3 = 18
7	x3 = 21
8	x3 = 24
9	x3 = 27

x4

0	x4 = 0
1	x4 = 4
2	x4 = 8
3	x4 = 12
4	x4 = 16
5	x4 = 20
6	x4 = 24
7	x4 = 28
8	x4 = 32
9	x4 = 36

x5

0	x5=0
1	x5=5
2	x5=10
3	x5=15
4	x5=20
5	x5=25
6	x5=30
7	x5=35
8	x5=40
9	x5=45

x6

0	x6=0
1	x6=6
2	x6=12
3	x6=18
4	x6=24
5	x6=30
6	x6=36
7	x6=42
8	x6=48
9	x6=54

x7

0	x7=0
1	x7=7
2	x7=14
3	x7=21
4	x7=28
5	x7=35
6	x7=42
7	x7=49
8	x7=56
9	x7=63

x8

0	x8=0
1	x8=8
2	x8=16
3	x8=24
4	x8=32
5	x8=40
6	x8=48
7	x8=56
8	x8=64
9	x8=72

x9

0	x9=0
1	x9=9
2	x9=18
3	x9=27
4	x9=36
5	x9=45
6	x9=54
7	x9=63
8	x9=72
9	x9=81

↑ GLUE TO 4X6 INDEX CARD ↑

Division Cross Out Sheet

÷0

÷0 IS NOT ALLOWED

÷1

1	÷1 = 1
2	÷1 = 2
3	÷1 = 3
4	÷1 = 4
5	÷1 = 5
6	÷1 = 6
7	÷1 = 7
8	÷1 = 8
9	÷1 = 9

÷2

2	÷ 2 = 1
4	÷ 2 = 2
6	÷ 2 = 3
8	÷ 2 = 4
10	÷ 2 = 5
12	÷ 2 = 6
14	÷ 2 = 7
16	÷ 2 = 8
18	÷ 2 = 9

÷3

3	÷ 3 = 1
6	÷ 3 = 2
9	÷ 3 = 3
12	÷ 3 = 4
15	÷ 3 = 5
18	÷ 3 = 6
21	÷ 3 = 7
24	÷ 3 = 8
27	÷ 3 = 9

÷4

4	÷ 4 = 1
8	÷ 4 = 2
12	÷ 4 = 3
16	÷ 4 = 4
20	÷ 4 = 5
24	÷ 4 = 6
28	÷ 4 = 7
32	÷ 4 = 8
37	÷ 4 = 9

÷5

5	÷ 5 = 1
10	÷ 5 = 2
15	÷ 5 = 3
20	÷ 5 = 4
25	÷ 5 = 5
30	÷ 5 = 6
35	÷ 5 = 7
40	÷ 5 = 8
45	÷ 5 = 9

÷6

6	÷ 6 = 1
12	÷ 6 = 2
18	÷ 6 = 3
24	÷ 6 = 4
30	÷ 6 = 5
36	÷ 6 = 6
42	÷ 6 = 7
48	÷ 6 = 8
54	÷ 6 = 9

÷7

7	÷ 7 = 1
14	÷ 7 = 2
21	÷ 7 = 3
28	÷ 7 = 4
35	÷ 7 = 5
42	÷ 7 = 6
49	÷ 7 = 7
56	÷ 7 = 8
63	÷ 7 = 9

÷8

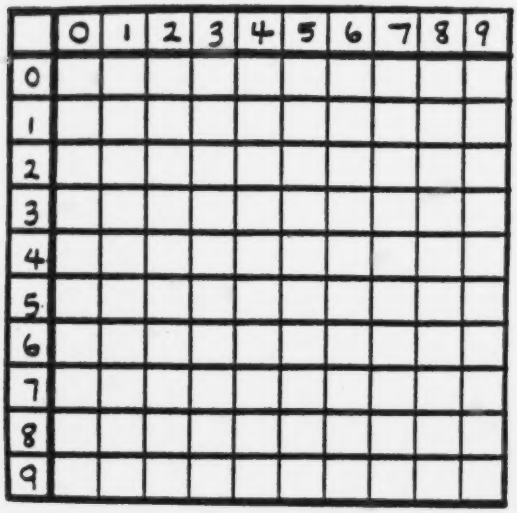
8	÷8 = 1
16	÷8 = 2
24	÷8 = 3
32	÷8 = 4
40	÷8 = 5
48	÷8 = 6
56	÷8 = 7
64	÷8 = 8
72	÷8 = 9

÷9

9	÷ 9 = 1
18	÷ 9 = 2
27	÷ 9 = 3
36	÷ 9 = 4
45	÷ 9 = 5
54	÷ 9 = 6
63	÷ 9 = 7
72	÷ 9 = 8
81	÷ 9 = 9

↑ GLUE TO 4X6 INDEX CARD ↑

An Alternative Recordkeeping Form for Teachers



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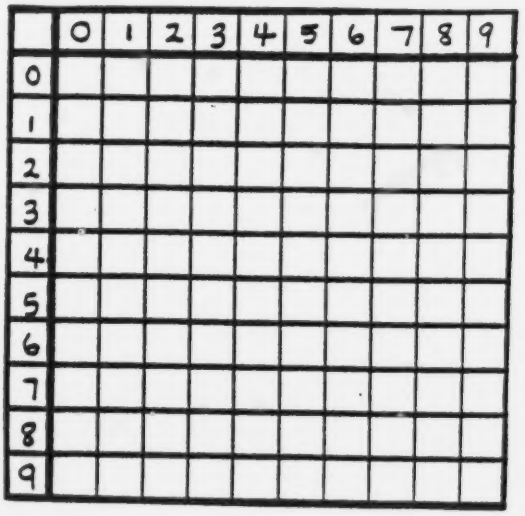

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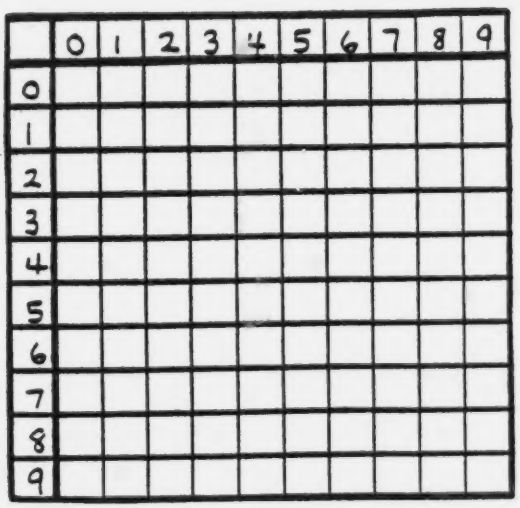

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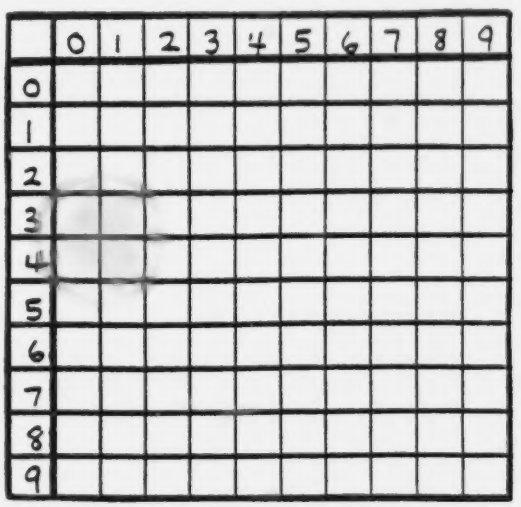

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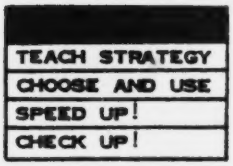


Look Ahead Activities . . .

. . . to teach or review important prerequisites.

(In the final draft, a commentary will follow the list for each operation.)

LOOK AHEAD-- to tomorrow, to three weeks from now. What basic facts are unknown and need to be memorized? Some students in our classes are "good memorizers" and need only be told to "study." We typically can anticipate that other students will have difficulty memorizing certain facts. Concept understanding and place value concepts and skills are assumed to be very important prerequisites to any memory work with facts. If teaching or reteaching is required, it is helpful to plan ahead so that when the time comes to help students learn strategies for remembering certain groups of facts, that teaching will flow smoothly. In addition to concept and place value work, systematically use sponge activities--which take only a minute or two--to review or teach important prerequisites to success in memorizing unknown facts.

For ADDITION, check:

1. Counting up skills (into teens).
2. Auditory recognition of (1, 2, 3) numbers said.
3. Visual imagery for 10-frame: "How many in the frame?" "How many needed to fill the frame?"

For SUBTRACTION, check:

1-3 above.

4. Counting back skills (from teens, too).
5. MASTERY OF RELATED ADDITION FACTS.

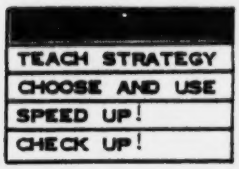
For MULTIPLICATION, check:

1. Mastery of clock times (to the nearest five minutes).
2. Sight recognition of finger patterns.
3. Ability to subtract a 1-digit number from a multiple of 10.
4. Given a 1-digit number, the ability to add another to "make 9."
5. Ability to double two 2-digit numbers mentally.

For DIVISION, check:

1-2 above.

3. MASTERY OF RELATED MULTIPLICATION FACTS.



Time And Again Activities

Most children benefit by repetition of certain basic activities, such as those described below. For obvious reasons, these activities are called "Time and Again" activities.

Counting Activities for Beginners

Looking Ahead to Easy Addition and Subtraction Facts

The early number program has its roots in counting. Indeed, to count with skill and understanding is an important problem solving skill in mathematics. Children refine and rely heavily on their ability to count in early school tasks.

Counting is the natural basis for solving addition and subtraction combinations in the beginning mathematics program, both within and apart from problem solving contexts.

When the concept of addition or subtraction is initially mastered by first grade students, the goal shifts to memorizing basic facts and more efficient counting schemes are desirable. An emerging thrust in school mathematics is to capitalize on the natural inclination of young children and encourage them to *count on* to solve addition problems like 4 + 2 or 3 + 8; to *count back* to answer subtraction problems 9 - 3 or 7 - 2. These number combinations involve "short counts" of one, two or three.

This thrust is a good one. *But there are hidden prerequisites to success.* It is helpful to *look ahead* to emphasize the following four extensions of counting:

- counting on;
- counting back;
- auditory patterning (especially for two and three counts); and
- visual patterning based on the 10-frame.

The goal is to assure that these counting skills are intact before we ask children to apply them in the early addition-subtraction program.

Starting about mid second semester in the kindergarten program, Look Ahead activities like those described below can be carried out *two to three times each week.* The emphasis at this level involves numbers to 10 or 12. This thrust is extended into the early first grade program and typically requires review at the beginning of second grade. Remedial programs treat these activities as needed.

Sample "Look Ahead" Activities

Counting On

Many excellent sources (e.g., Lorton, (1976); Leutzinger and Nelson (1979); Rathmell (1978); Richardson, (1982); Thornton and Toohey (1984) describe activities to help children count on from a given number. Introductory activities use countable objects. Later the transition to numbers is made. The immediate goal is to provide many opportunities in which the first vocal number in a count is *not* "one." Start with

any other number, 3 - 9, and count on two or three more.

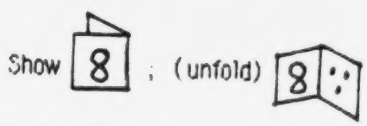
Time and Again Activity 1

* *Wee Handful* (Materials: 12 small counters in a box with lid.)
A child takes a "wee handful" (e.g. 5) from the box, counts how many are taken, and places them into the lid of the box. Hold the box lid high, so no one can peek. "Let's put more counters into the lid. Count with me." "How many in the lid?" (5) "Now. . . "(as, one by one, three additional counters are dropped into the lid) ". . . there's 6, 7, 8."

If it seems helpful, allow children to recount from 1 to check. Then return all counters to the box and repeat for other "wee handfuls." Each time, get children to *count on aloud* from the number of counters in the lid as, one by one, one, two or three additional counters are dropped into the lid.

Time and Again Activity 2

* *Say, Unfold and Count On!* (Materials: Folded number/dot cards as shown.) Use numbers 3-9 and one, two or three dots on the cards.)



Children say: "8, . . . 9, 10, 11."

Repeat for other cards (3-9) and one, two or three dots.

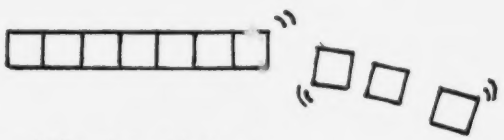
Counting Back

Many primary level children find it difficult to count backwards, even for short counts. It is a skill deserving greater emphasis in the early number program.

This emphasis can be carried into the home. Children can count the toys in the bath, then count backwards as they take them out. In one home a child counts forwards as she gives one parent "good night hugs," then counts backwards as she hugs the second.

Lorton (1976) describes many excellent activities for counting backwards. A simple beginning activity many teachers like is described below.

Time and Again Activity 3



* *Break Off!* (Materials: 16 unifix cubes).

Taking turns, children make a unifix cube train (e.g., a "9" train) and count backwards as one by one, they break off cubes from the train. Special emphasis for numbers 5-12 is on breaking off/counting back *two* or *three* .

Example: "How many cubes in this unifix train?" (9--It's all right for children to count with you to determine the total number.) " I'll break off the three white ones one at a time and give them to Alice. Count backwards with me as I do it. How many here?" (Indicate all cubes in the train: 9.) *Break off one:* "Now there's ?" (8). *Break off another: '* Now there's ?" (7) *Break off the third: 'Now there's* ?" (6). "Yes: 9--8,7,6. Six cubes still in the train."

Note: Some children can count backwards from 10 but not from 9 or 13. Starting at 16, many children omit 15 or 13 when they count backwards. Even though the direct application of counting backwards generally is restricted to numbers 12 or less in early subtraction, it is useful to start beyond 15 from time to time with first and second grade students.

Auditory Patterning

Is it necessary for children to finger count as they count on to add or count back to subtract 1, 2 or 3? The following activities help students internalize these short counts and thus make finger counting unnecessary.

The activities sensitize children to the auditory pattern which accompanies the beat of two vs three claps; the hearing of two vs three numbers.

Time and Again Activity 4

* *How Many Claps Do You Hear?* (Materials: None.)
  Clap one, two or three times and ask students to tell how many claps they heard.

Note: When children become comfortable with this first activity, proceed to that which follows.

Time and Again Activity 5

* *How Many Numbers Do you Hear?* (Materials: None.)
  Say two or three numbers in sequence (forwards or backwards), and ask students to tell how many numbers they heard.

  Examples: "*6, 7, 8.*" ("Three numbers.")
  "*10, 9.*" ("Two numbers.")

Note: This a *Time and Again* activity. The payoff comes later, in the addition-subtraction program. Given 5 + 3, for example, students will tend to recognize (without finger counting) when they've counted on **three** more: "5, -- *6,7,8.*" Or, for problems like 11 - 2, they will tend to recognize (without finger counting) when they've counted back **two**: "11, -- *10,9.*" The following Time and

Again activities involve body movement and further reinforce the auditory patterning for two or three.

Time and Again Activity 6

* *Say and Stand* (Materials: None.)
  Sit, hands on knees. Say a number (e.g. 8). *Count on two* before you stand. Children mimic. Repeat for other numbers.

  When children become comfortable with the "two more" pattern, carry out a similar activity in which the challenge is to count back two before you stand: "8, -- 7, 6."

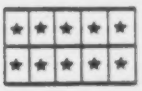
Time and Again Activity 7

* *Touch Your Toes* (Materials: None.)
  Stand, both hands above your head. Say a number (e.g. 9). Count on three before you touch your toes. Children mimic. Repeat for other numbers.

  When children become comfortable with the "three more" pattern, carry out a similar activity in which the challenge is to count back three before you touch your toes.

Visual Patterning Based on the 10-Frame

Good visual imagery for the 10-frame is a very important skill. One example of ten things, or six or eight--is that number of objects "in the frame." (Wirtz, 1980) If students have their own individual 10-frames, one suggestion is to have them "fill" the frame *left to right*, to follow the left-right reading motion.



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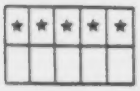
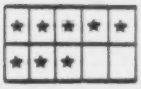
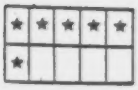


5 (top row filled)



8 (count on from 5 in top row)



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Two basic activities can be used to build visual imagery for the 10-frame:

Time and Again Activity 8

* *10-Frame Flash.* (Materials: A set of 10-frame cards [p. 154].)
Flash each card briefly, then remove it from sight. "How many stars did you see?" Repeat, emphasizing 6 - 9 stars in the frame. It often is helpful to make comments like: "Nine stars -- that must be *all but one* filled. Let's check." "Seven stars--that must be the top row and two more: 5,-- 6,7. Let's check."

Time and Again Activity 9

* *Frame Fill.* (Materials: 10-frame cards, as above.)
Flash as above, then ask children to close their eyes. "Can you see the stars in the frame? How many would it take to fill the frame?... Let's check."

Note: Good mental imagery for the 10-frame paves the way for direct success later with 10 sums and subtracting any number from 10. It also is basic to using 10 as a bridge for adding facts like 6 + 8 and 7 + 5; and for subtracting facts like 13 - 5, 12 - 4.

Other Time and Again Activities

Visual Recognition of Finger Patterns

The following activity for "Beginners" would be used as needed, only with students who are finger counting. If they "must" use fingers, the goal is to help them use fingers in an efficient manner. If the above counting skills are developed and other strategies and sequences suggested in this book are used, children typically have no need to count on their fingers.

The variation for "Older learners" is useful if you intend to introduce finger patterns for Pattern 9s (p. 112). The activity suggested below would be started three to four weeks before this multiplication lesson. Ideally, it would be carried out two to three times each week until multiplication 9s have been introduced.

Time and Again Activity 10

* *Quick Fingers*

  Beginners: Flash fingers to see if students can recognize five fingers (one hand); four (all but one finger on a hand); six (a hand and one extra finger); and other finger patterns.

  Correlate with time on the hour. As you move the hour hand of a demonstration clock between the numbers 1 to 10, ask students to *use fingers* to tell time on the hour. Be sure children "use up" all fingers on one hand before using finger from the other hand.

  Older Learners: Use subtraction flash card drills to prepare for the finger pattern approach to multiplication 9s. Have students use fingers to show answers. Instruct them always to "use up" the fingers on one hand before using the other hand. For 12-5, for example, students would hold up five fingers on one hand, two fingers on the other.

Add/Multiply to Check Activities

The following activities help build and reinforce the addition/subtraction and multiplication/division relationships. Explicit activities like those outlined below also increase the potential of students using addition or multiplication facts they know to answer unknown subtraction or division facts. For other activity suggestions, see p. 64 (for subtraction) or p. 102 (for division).

Time and Again Activity 11

* *Break a Stick.*

Prerequisites:
- Firm mastery of related addition facts; and, for easier subtraction use unifix cubes to model the subtraction facts.
- Instruction based on the five-phase activity sequence of p. 5.

Use unifix cubes to model the subtraction fact.

Example for 7 - 2: Make a train of 7 cubes, break off 2.
Then put the parts back together: add to check.
"7-2=5 because 2+5=7."

Write the subtraction fact and its "add to check" fact.

Time and Again Activity 12

* *Share!*

Prerequisites:
- Firm mastery of related multiplication facts; and, for easier division facts,
- Instruction based on the five-phase activity sequence of p. 5.

Use unifix cubes to model the division fact.

Example for 12 ÷ 3: Share 12 cubes among 3 children.
Then put the cubes back together: multiply to check.
"12÷3=4 because 3x4=12."

Write the division fact and its "multiply to check" fact.

Time and Again Activity 13

* *Write Fast!*
Show a subtraction card. Students write the problem (with answer) and its "add to check" fact on their slates. On worksheets, students can select three favorite subtraction facts and, for each, write the "add to check" fact beside it.

From time to time, ask students to use counters to illustrate the subtraction and its "add to check" fact.

Similarly, students can write "multiply to check" facts for given division problems. From time to time, ask students to use counters to illustrate the division and its "multiply to check" fact.

Activity to Reinforce the Commutative Relationship

After one group of addition or multiplication facts is introduced, the turnarounds (commutatives) of these facts also would be studied. Start with countable objects as in the addition "Turn Around!" activity (p. 40) or the multiplication "Same Answer, Both Ways" activity (p. 82).

Regularly repeat the activity which follows.

Time and Again Activity 14

* *Find Your Turnaround* (e.g. 2 + 9, 9 + 2; 2x7, 7x2)
Give each student an addition (multiplication) basic fact card. Each child finds the person holding the turnaround card. Then students write the two facts side by side on the board for others to and complete. If a set of cards is marked (e.g. with a paper clip), children should use objects or array cards to illustrate that both have the same answer.

Activity to Reinforce the Partner Relationship

When children have learned one group of subtraction or division facts *well*, the related partner facts would be studied. Note that for subtraction count Backs, it is suggested that this delay be prolonged. In this teacher reference, the Count Back

lesson is on p. 56. The lesson for Count Ups--the partner facts--is delayed until p. 70.

Time and Again Activity 15

* *Find Your Partner* (e.g. 9-2, 9-7; 36+9, 36+4)
Prerequisite: Activity 1 (p. 67, subtraction; p. 105, division)

Give each student a basic fact card for subtraction/division. Each child finds the person holding the partner card. Then students write the two facts side by side on the board for others to copy and complete.

Activities to Consolidate Known Facts

When students have studied both a fact and its turnaround or partner fact, then fact families can be introduced.

Time and Again Activity 16

* *Join Your Family* (e.g. 9-2, 9-7, 2+9, 9+2; 36+9, 36+4, 9x4, 4x9)
a. Extend "Find Your Partner", Activity 15, above, to include all facts in a family.
b. Give each child a fact card. Say, "I'm calling the family for 9, 2 and 7 (or 36, 9 and 4 or...)" Students holding the appropriate facts stand, read and answer their fact and, if correct, trade their card with a friend.

Extensions of Basic Facts

Time and Again Activity 17

* ***Use Objects to Check*** (Reinforce concepts)
On a regular basis, have students circle one or two facts on a worksheet and use objects or draw pictures to show their anwers make sense.

Time and Again Activity 18

* ***What's the Story?*** (Integrate problem solving)
On a regular basis, have students circle a fact on a worksheet, turn the sheet over and:
  - draw a picture of a situation that uses numbers in the problem.
  - write (tell) a story using the numbers.

Teach Strategy:
What Will Help?

Help students master unknown facts. Explicity teach strategies which make memorization easier/possible. Planned activities and follow up exercises help! The following strategies have proved helpful to many students. The lessons beginning on p. 38 teach these strategies.

For ADDITION:
1. **Start big and count on** (for +1, +2, +3 facts: 2+6, 9+3...).
2. **Use the turnaround** (8+3=11, so 3+8=11...).
3. **Think of the picture** (for doubles: 4+4, 7+7...).
4. **Think of a double** (doubles + 1 like 4+5, 5+6, 6+7, 7+8).
5. **Think of the 10 frame** (especially for 6+4, 4+6).
6. **Use the pattern** (for 9s: 9+5, 7+9...)
7. **Make 10, add extra** (especially for 7+4, 7+5 8+4, 8+5, 8+6...).
8. **Use an addition fact I know** (e.g. 9+5=14, so 8+5=13 [1 less]...).

For SUBTRACTION:
1. **Use an addition fact I know** (e.g. 8-3=5 because I know 3+5=8...).
2. **Count back** (for -1, -2, -3 facts: 11-2, 9-3,...).
3. **Count up** (when "numbers are neighbors" [look close]: 11-9, 9-6,...)
4. **Use a doubles picture** (12-6, 8-4...).
5. **Think of the 10-frame** (especially for 10-6, 10-4).
6. **Use the partner fact** (I know 9-2=7, so 9-7=2...).
7. **Use my "0" finger** (for teen minuend facts like 13-5, 15-7, 14-6,...).
8. **Use a subtraction fact I know** (10-7=3, so 11-7=4 [1 more]; 12-6=6, so 13-6=7 [1 more].)

For MULTIPLICATION:
1. **Use the picture** (for 2s: 8x2, 2x6...).
2. **Use the turnaround** (2x7=14, so 7x2=14...).
3. **Use the clock** (for 5s: 7x5, 5x4...).
4. **Use finger or number patterns** (for 9s: 9x6, 4x9...).
5. **Think half--and double it** (4x6? just double 2x6...).
6. **Use an easy multiplication fact** (9x4=36, so 8x4=32 [4 less]...)
7. **Use other "One a Week" Ideas** (see p. 84)

For DIVISION:
1. **Use a multiplication fact I know** (64+8=8 because I know 8x8=64...).
2. **Use the picture** (for 2s: 14+2, 16+2...).
3. **Use the clock** (for 5s: 40+5, 20+5...).
4. **Use finger of number patterns** (for 9s: 63+9, 27+9...).
5. **Use the partner fact** (I know 35+5=7, so 35+7=5...).

Facts Grouped By Strategy For Recall*

100 Addition Facts (Remember "Turnarounds")

Easy Facts

- Count Ons: (45**) 1 + (1,2,3,4,5,6,7,8,9); 2 + (2,3,4,5,6,7,8,9); 3 + (3,4,5,6,7,8,9)
- Zeros (19): 1 + (1,2,3,4,5,6,7,8,9)
- Doubles (6) : 4+4, 5+5, 6+6, 7+7, 8+8, 9+9
- Easy 10-Frame (4): 6+4, 5+4

Magic 9s (10): 9 + (4,5,6,7,8)

Harder Facts

- Doubles +1 (6): 5+6, 6+7, 7+8
- Make 10, Add Extra facts: (10) 7 + (4,5); 8 + (4,5,6)

100 Subtraction Facts

Easy Facts

- Count Backs (27): (11,10,9,8,7,6,5,4,3) - 2; (12,11,10,9,8,7,6,5,4) - 3; (10,9,8,7,6,5,4,3,2) - 1
- Zeros (19): n-n, n-0 (n = any number, 0 through 9)
- Doubles (6): 18-9, 16-8, 14-7, 12-6, 10-5, 8-4
- "Easy 10-Frame" facts (5): 10 - (9,8,7,6,4) Also (2): 9-5, 9-4
- Count Ups (15): (12,11) - 9; (11,9) - 8; (9,8) - 7; (9,8,7) - 6; (8,7,6) - 5; (7,6,5) - 4

Harder Facts

- Addition Will Help/Zero Finger facts (26): (17,16,15,14,13) - 9; (17,15,14,13,12) - 8; (16,15,13,12,11) - 7; (15,14,13,11) - 6; (14,13,12,11) - 5; (13,12,11) - 4

Note: There are patterns for subtraction 9s. And, for some facts, children may use known subtraction facts to help.

*Once facts are introduced in Teach Strategy and Choose and Use Activities, include them systematically in daily Call Up activities.
**For addition facts, includes turnarounds.

100 Multiplication Facts (Remember "Turnarounds")

Easy Facts

- Picture 2s (15*): 2 x (2,3,4,5,6,7,8,9)
- One a Week Facts: (15) 6x7, 3x4, 7x8, 3x3, 3x7, 6x4, 6x6, 6x8, 8x8
- Clock 5s (13): 5 x (3,4,5,6,7,8,9)
- Zeros (19): 0xn (n = any number, 0 through 9)
- Ones (17): 1xn (n = any number, 1 through 9)
- Pattern 9s (11): 9 x (3,4,6,7,8,9)

Harder Facts

Twice as Much Facts (9):
4x(4,7,8); 8x3, 6x3

Last Fact

Miner Fact: 7x7

90 Division Facts

Easy Facts

- Picture 2s (15): (18,16,14, 12, 10, 8,6,4) ÷ 2 ----> Partners: 18÷9, 16÷8, 14÷7, 12÷6, 10÷5, 8÷4, 6÷3
- Clock 5s division (13): (45,40,35,30, 25, 20, 15) ÷ 5 ----> Partners: 45÷9, 40÷8, 35÷7, 30÷6, 20÷4, 15÷3
- Pattern 9s (11): (81,72,63,54,36,27) ÷ 9 ----> Partners: 72÷8, 63÷7, 54÷6, 36÷4, 27÷3
- Zeros (9), Ones (17): O÷n; n÷n =1; n÷1=n (n = any number, 1 through 9)

Harder Facts

- Squares (5): 64÷8, 49÷7, 36÷6, 16÷4, 9÷3
- "Multiply to Help" facts (20): (56,48,32,24) ÷ 8; (56,42,28,21) ÷7; (48,42,24,18) ÷6; (32,28,24,12) ÷ 4; (24,21,18,12) ÷ 3

*For multiplication facts, include turnarounds.

Teaching Sequences for Addition and Subtraction Facts*: Overview

(The commentary in this outline will be expanded in the final draft.)

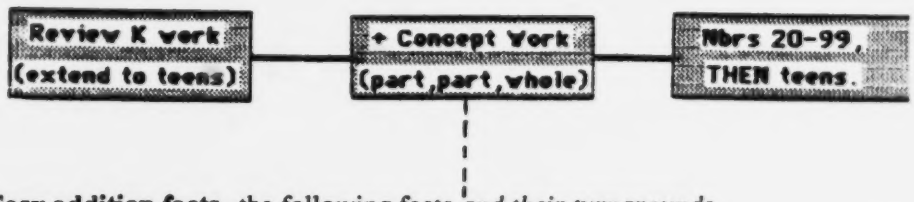
Grade K (Numbers 0-10 or 12):

At this level, focus on four counting skills provides a strong basis for number fact work in grade 1:

- Counting On
- Counting Back
- Auditory patterning (How many numbers did you hear?)
- Visual Patterning (How many fingers do you see? How many objects are needed to fill the 10-frame?)

Grade 1: Suggested Sequence

1. **Early Work (start with 0 - 12)**



2. **74 Easy addition facts**--the following facts *and their turnarounds* (about two months):

   - **Count Ons** (45 facts: 2+7, 8+3, 1+4...)
   - **Zero facts** (19 facts: 5+0, 0+3...)
   - **Doubles** (6 facts: 4+4, 9+9...)
   - **Easy 10-Frame facts** (4 new facts: 6 + 4, 4 + 6, 5 + 4, 4 + 5.)

   ] Any order

   Cross off facts as children learn them (see p. l3).

   The topics might be presented "between" fact groups any time when a "break" in number fact work seems useful. Place value work is most important and can be treated earlier than some textbooks place it. It often is necessary to turn to other curricular topics to allow students time to learn one group of facts well before introducing another.

**The sequence of the school adapted mathematics textbook/program otherwise would be followed.*

- place value (20-99, then teens)
- time
- money
- measurement
- graphing
- geometry
- fractions
- problem solving (on-going)

3. **Subtraction concept** (about the third or fourth month of school).

4. **Easy subtraction facts** *(Major number fact focus for second semester).*

   For each fact group below, *first:* Teach efficient recall strategy. *Later:* add to check. *Eventually:* study partners and families of facts.

   - **Count Backs** (27 -1, -2, -3 facts: 8-3, 11-2, 9-1...)
   - **Zero facts** (19 facts: 0 - 0, 1 - 1, 1 - 0, ...9 - 9, 9 - 0)
   - **Doubles** (6 facts: 12-6, 16-8...)
   - **Easy 10-Frame facts** (5 facts: 10-4,10 - 6, 10 - 7, 10 - 8, 10 - 9)

   ] Any order

   - **Count Ups** (15 facts: 9-7, 11-8, 12-9, 8-5, 7-6, and other facts in which the numbers are "close neighbors." Teachers will note that "good" Count Ups are those for which the count is short. That is, facts having *answers* of 1, 2 or 3. See p. 70.)

   Maintain short, daily drills on addition facts throughout second semester.

5. Mid to late spring (if not studied earlier): Magic 9s in addition.

6. Only if above is mastered and children <u>don't need to count</u> for these "harder" facts:

   - Doubles +1 facts (5+6, 6+7, and 7+8); "Make 10, Add Extra" facts (7+4, 7+5, 8+4, 8+5, 8+6)--and turnarounds.
   - Other subtraction facts.

Grade 2: Suggested Sequence

1. Review 0-12, emphasizing:

   Counting on (to 15); counting back (from 15 or less);* "How many numbers do you hear?" (two or three numbers--up or back); 10-frame flashes (two or three times a week throughout the first semester.)**

2. Review addition concept (part, part ;whole) and "clump strategies" on these easy addition facts:

   • +9s and doubles (first)
   • Count ons and zeros
   • 10 sums (esp. 6+4); 5+4

   ] and turnarounds

   Integrate with other curricular topics to give students time to master addition facts well before turning to subtraction:

• Place value (2-digit numbers)	
• Money	• Measurement
• Time	• Geometry
• Place value (3-digit numbers)	• Fractions
• Problem solving (on-going)	• Graphing

3. Review subtraction concept (whole, take away part, leaves part) and easy subtraction facts (see p. 29). Continue 10-frame flashes and drill on easy addition facts. Many teachers introduce 2-digit addition without regrouping and the curricular topics boxed above "between" fact groups. This gives children extra time to learn one set of facts well before presenting another.

---

*Continue until all Count Backs are mastered.
**Continue until all Count Backs and Count Ups are mastered.

4. By about the third month of school, focus on the remaining addition facts:

   - Doubles +1 (3): 5+6, 6+7, 7+8
   - "Make 10, Add Extra" facts (5): 7+4, 7+5, 8+4, 8+5, 8+6

   ] and turnarounds

   Integrate systematic number fact work with other curricular topics, as above. Include 2-digit subtraction without regrouping. Continue daily drills on addition and easy subtraction facts.

5. <u>During Second Semester</u>. Continue daily drills based on the 100 addition facts and easy subtraction facts.

   <u>Major Focus</u>: Other subtraction facts not yet treated. Help students answer *unknown* facts by using addition facts they know, number patterns (especially for -9s) or other relationships (e.g. one more than a "10" or a double: 10-7=3, so 11-7=1 more (4); 12-6=6, so 13-6=1 more (7)). Try the "Zero Finger" approach (p. 76) near the end of the semester with students who still have difficulty with harder subtraction facts.

   - Treat 2-digit addition with regrouping.
   - Do 2-digit subtraction with regrouping before spring testing. Allow use of Cross Out list as needed.

Grade 3 and Beyond: Suggested Sequence

1. As needed, to review: counting back (from 15 or less); "How many numbers do you hear?" (two or three--up or back); 10-frame flashes.
2. Review the addition concept and easy addition facts (see p. 29). Start with 9s and doubles. "Clump" strategies. That is, work with two different fact groups at a time--one near the beginning, another near the end of a math period. Turn to the curricular topics listed below as needed so students have time to learn these facts well before others are introduced.

- Place value (2,3,4 digit)
- Time (needed for Clock 5s)
- Money
- Graphing (on-going)
- Estimation/Mental Computation (on-going)
- Measurement
- Geometry
- Fractions
- Problem Solving (on-going)

3. Review the subtraction concept and easy subtraction facts (see p. 29). Continue 10-frame flashes and drill on easy addition facts. Integrate fact work with other curricular topics (above). Include 2-digit or 3-digit addition without regrouping.

4. Review the "harder" addition facts:

Three Doubles +1: 5+6, 6+7, 7+8

Five "Make 10, Add Extra" facts: 7+4, 7+5, 8+4, 8+5, 8+6

] and turnarounds

- Continue systematic review of addition and easy subtraction facts and time on the clock.
- Emphasize place value work and include addition and subtraction (two and three digits) without regrouping.

5. Review the "harder" subtraction facts. Help students answer unknown facts by using addition facts they know, number patterns (especially for -9s) or other relationships (e.g. one more than a "10" or a double: 10-4=6, so 11-4 is one more (7); 14-7=7, so 15-7=1 more (8)). Try the "Zero Finger" technique, p. 76, with students who cannot succeed with these approaches.

- Consider using the Cross Out list (p. 7), especially with slower students.
- Continue daily addition and subtraction fact drills.

6. Treat addition/subtraction with regrouping.

Note: Early work with multiplication could be started before students complete the addition/subtraction computation parts of the year's curriculum.

Using the Sample Lessons

(The final draft will include a one-page introduction to this section. Depending on the students, a "lesson" could take two days or two weeks. The idea of "clumping" strategies for older students will be addressed.)

Count Ons

Verbal Prompt: "See a 1, 2, 3 in addition?
Start big and count on."

There are 45 Count Ons: The +1 Count Ons: 1+1, 2+1, 3+1,... 9+1;
the +2 Count Ons: 1+2, 2+2, 3+2,... 9+2;
the +3 Count Ons: 1+3, 2+3, 3+3,... 9+3;
and their turnarounds (commutatives).

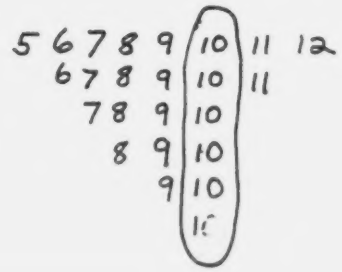
[redacted] **Count On activities:**

* *Draw More.* (Materials: Classroom chalkboard.) Draw three small circles on the board. "How many circles do you see?" (3) "Let's draw more. Count on as I do it." *Cover the first group of three:* "How many here?" (3) *Get children to count on with you, aloud, as you draw more circles:* "...4, 5, 6." Erase and repeat. Each time, draw several circles; count them; then cover, and get children to count on as you draw two or three more.

* *Wee Handful.* (Time and Again Activity 1, p. 19)

* *Say, Unfold and Count On!* (Time and Again Activity 2, p. 19)

* *Six of a Kind.* Materials: Number cube labeled 4, 5, 6, 7, 8, 9; cube with one, two or 3 dots on each face.) This is an independent activity for two or three students. Students roll the cubes, say the number, count on one, two or three and record the total. The first to get six of the same number loops them, and wins the round.



Auditory patterning activities:

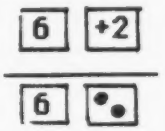
* *How Many Claps Do You Hear?* (Time and Again Activity 4, p. 21)
* *How Many Numbers Do You Hear?* (Time and Again Activity 5, p. 21)
* *Say and Stand.* (Time and Again Activity 6, p. 22)
* *Touch Your Toes.* (Time and Again Activity 7, p. 22)

---

* *How many now?* [Materials: 9 unifix cubes (any color), 3 additional cubes (different color)]. Make a train of 5 cubes. "Five here. Count on with me as I add more cubes to the train." Attach three extra cubes, one at a time, as children count: "5,....6, 7, 8. As the children to write the addition fact to match (5 + 3 = 8). Repeat for other Count Ons.

[redacted]

* *Draw and Say.* (Materials: Classroom chalkboard; +2 card as shown (+2; on front, 2 dots on the back); similar cards for +1 and +3.) Write a number (2-9) on the board and hold a card (+1,+2, or +3) beside it. After children say the greater number, flip the card as they continue the count. For example, "6, ...7,8." (Repeat for other numbers.)



Note: If you are working with developmentally slow or learning disabled students, it may be helpful to focus just on the +2 count ons before including the +1 or +3 facts.

[redacted]

* *After Me.* (Materials: Flash cards for Count Ons.) As you show each card, have students say the greater number. Continue the count and say the sum. Then switch roles. You say the greater number and have students count on and say the sum.

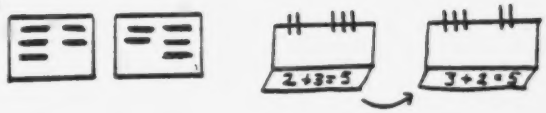
(This is a very important activity in which the teacher models the counting on.)

* *I Spy!* (Materials: Flash cards for Count Ons; heavy paper for glasses.) Help students make a monocle or magnifying glass out of heavy paper. Let students raise/look through their glasses whenever they SPY a Count On among cards you present in a flash card drill.

  Each time call on one child to count on aloudto the sum. Count with the child, if necessary.

Turnarounds

* *Turn Around!* (Materials: Lego board or clothespin card, as shown.) Turn a lego board on a card with clothespins on card to illustrate that, in addition, turnarounds (commutatives) have the same answer.



* *Like You Best!* (Materials: Workpage of addition facts.) On any math fact worksheet you may assign, have students circle three "favorite" facts. Instruct them to do these first and to write the turnaround of each beside it.

  As you walk about the room, ask students individually to use counters you provide to show that, in addition, turnarounds have the same answer. One of the three circled problems can be used to illustrate.

---

* *Find It, Chalk It!* (Materials: Flash cards for Count Ons mixed with others; individual chalkboards.) Children write and show answers just to Count Ons they see. Students can draw a tally box on their boards and give themselves a tally for each correct answer.

[redacted]

* *10 or More.* (Materials: Flash cards for Count Ons.)
As you flash the cards, children hold thumbs up if the answer is 10 or more, otherwise hold thumbs down. *Each time, call on one student to give the answer:* "Yes, thumbs up because the answer is . . ."

• Worksheets: Have students find and write answers just to the Count Ons they find. If 9+3, 2+9, 8+3 or some other Count On isn't on a page, write it on the board so students can copy and complete it in the margins of the page.

[redacted]

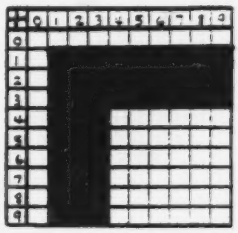
FOCUS ON the following facts and their turnarounds:

The +1 Count Ons: 1+1, 2+1, 3+1, ... 9+1;

the +2 Count Ons: 1+2, 2+2, 3+2, ... 9+2;

the +3 Count Ons: 1+3, 2+3, 3+3, ... 9+3.

...and other facts previously studied.



Zero Facts

Verbal Prompt: "A zero in the problem makes no change."

There are 19 Zero facts: 0 + (0,1,2,3,4,5,6,7,8,9) and their turnarounds.

These facts tend to be very easy for children. A firm mastery of the addition concept and related number work are adequate prerequisites. The concept of addition is reinforced in the following activities.

---

* *On and Off.* (Materials: 10 small counters for each child.) Have students place their math books to one side on top of their desks. Set a few counters on top of each child's book and a few more off to the side on the desk. Ask the students to tell how many counters are <u>on</u> the books, how many are <u>off</u> the books, and how many there are in all. Remind students by frequently placing <u>no</u> counters on (or off) a book that "+0 makes no change."

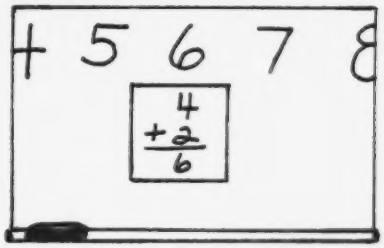
---

* *Snap!* (Materials for every 7-8 students: 10 small counters and a deck of Zero and Count On cards in a paper sack.) Arrange each group of 7-8 children in a circle and let them begin to pass one bag. Snap at random to stop the pass. At the snap, each child holding a bag draws a card and gives the answer. If it is a Zero fact, the student points to someone in the group who then uses counters to show that the answer given is correct.

* *Show It!* After students complete all Count Ons and Zeros on worksheets, have them circle and use counters for one or two Zero problems to verify the answers they have written.

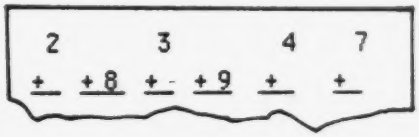
---

* *Match and Write.* (Materials: Classroom chalkboard; flash cards for Count Ons and Zeros.) Write the numbers 0 - 12 on the chalkboard. As children enter the room (after lunch, recess...), they take the flash card you give them, write it on the board near its answer, and return it to the flash card box on your desk. Then, at their seats, they copy problems from the board (with answers) until you signal stop and collect papers.



* *Try to Beat!* See Game Directions for Parents, p. 133.

* *Race Row.* (Materials: A ditto like that shown on which just one addend is given.) Distribute the papers and say, "The race row number for today is--0 ! (or 1 or 2 or 3)" At this signal students write the given number into each problem and race to finish the row. Some teachers like to let students race against the clock rather than against each other. Allow no more than 3-4 seconds per problem, and call time at the end. Grade the problems together and let students put a star or sticker beside each row which was correctly completed before you called time.

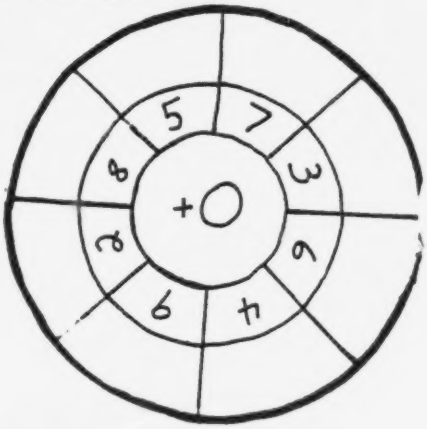


Helpful hint: Make a whole page of problem rows like that shown. Use just one row a day, then collect and reuse a different time. This game is good for Count Ons, Zeros, and Doubles. In the latter case, students merely double the given number.

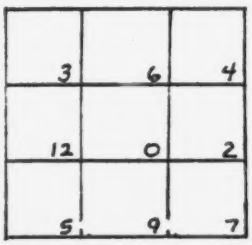
* *Telephone Sums.* (Materials: for each pair of students: a page from an old telephone book; lined or square centimeter paper, pencils and a number cube or spinner labeled 1, 3, 0, 2, 3, 0.)
Let children pair up. One child selects a telephone number from the page which each copies. The other student rolls the cube/spins the spinner and both children race to add this number (e.g. 0) to each telephone digit as shown. The child who finishes the round first (with correct answers) earns one tally point. Play until teacher calls time.

+	4	5	2	8	6	3	6
0							

* *Hot Wheels.* (Materials: For each student, a laminated number wheel like that shown; crayon and wipe off rag.)
Write +0 (or +1 or +2 or +3) on the board for children to copy into the center of their wheel. Then, at "Go!" they race to finish before you ring a bell/ flick the lights. Grade the wheels together. Have students clean their wheel and pass it to another student (e.g., the person behind you; to your left...). Play again with a new wheel.

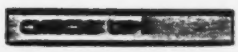


* *Three in a Row.* (Materials: Flash cards for Count Ons and Zeros; individual chalkboards.) To start, children write a number (0-12) in the lower corner of each square. These numbers are answers to flash cards used in the activity. As the cards are flashed, children copy problems in the appropriate answer squares. If a square already has a fact in it, another is not written. The first child to get three in down, across, or on



Game Mat

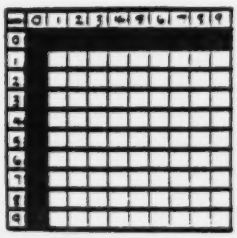
a diagonal calls, "THREE IN A ROW!" The child rereads each problem and its answer while others check. If correct, a tally point is earned. Children draw another game mat and play resumes. See who wins the most points before time to stop.



FOCUS ON the following facts and their turnarounds:

0 + (0,1,2,3,4,5,6,7,8,9)

...and other facts previously studied.



Easy 10-Frame Facts

Verbal Prompt: "Fill the 10-frame."

There are 4 new facts to be emphasized: 6+4, 4+6, 4+5 and 5+4 and their turnarounds. In preparation for harder addition and subtraction facts, also treat, in the frame: 5+5, 1+9, 2+8, 3+7 and their turnarounds.

[redacted]

Activities to promote visual imagery for the 10-Frame:

* *10-Frame Flash.* (Time and Again Activity 8, p. 23)

* *Frame Fill.* (Time and Again Activity 9 , p. 23)

[redacted]

* *In the Frame.* (To be written. Consider giving students individual 10-frames [p. 154] and chips for this activity. Students use the 10-frames to answer to written 10-sums facts.)

* *Do and Check.* (To be written. Children write answers to 10-sums, then use a 10-frame [p. 154] to check.)

* *All But One.* (To be written. 5+4: That's "all but one" filled in the 10-frame.)

[redacted]

* *Thumbs Up!* (Materials: Flash cards for 10-sums and other known facts.) As you flash the cards, children should hold thumbs up for 10-sums, otherwise thumbs down. Provide feedback like: "Yes, 6 and 4 more fills the frame; 6+4=10. Try to let 4+6 and 6+4 appear more frequently than other facts in the deck.

[redacted]

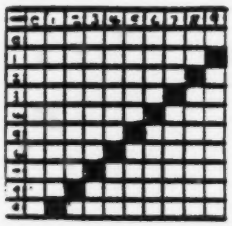
* *Pop Up!* (Materials: Flash cards [for teacher]; enough cards, numbered 5 to 12, to give one to each child. If possible, have at least two cards for each number. It is good to have even more cards for "10" and for other larger numbers, especially 9 -12.) As you flash and read each fact card, students holding the correct answer "pops up" (stands). At your signal they trade cards with a classmate (before you count to 5).

[redacted]

FOCUS ON the following facts and their turnarounds:

6+4, 4+6, 4+5, 5+5

...and other facts previously studied.



Doubles

Verbal prompt: "Picture the double."

There are 6 Doubles to be emphasized: 4+4, 5+5, 6+6, 7+7, 8+8, 9+9.

[redacted]

These facts tend to be very easy for children. A firm mastery of the addition concept and related number work are adequate prerequisites. The concept of addition is reinforced in the work with pictures.

[redacted]

* *Picture Doubles*. Use the 9+9 "Two Loops" fact to illustrate the technique of using pictures [p. 161] to help children learn addition doubles.

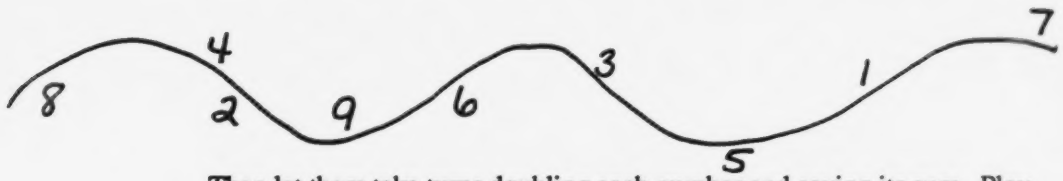
[redacted]

* *Scramble* (Materials: For every 8 students, one set of pictures and cards for Doubles [p. 161]. Starting with 4, assign each student an even number 4 - 18]. Repeat as necessary until all students are numbered. In clusters of eight [4 - 18] on the floor, students mix the doubles cards and turn them face up. At "go", each child scrambles to find the Doubles picture and fact that match the number assigned.

* *One and Only!* (Materials: Deck of small Doubles cards: facts and their related pictures [p. 171].) Use "Old Maid" card rules. Alternatively, omit the "One and Only" card and play "Concentration" using familiar card game rules. Make matches: the double to its related picture, and say the answer.

[redacted]

* *Double It!* (Materials: Individual chalkboards or paper and pencil). Let students work in pairs. Have them copy the following from the board:



Then let them take turns doubling each number and saying its sum. Play short, one - minute rounds. Players can keep tally of the number of Doubles sums they give in the time allowed. Variation: Taking turns, players randomly point to numbers to be doubled by the other player .

Note: The above work would be extended to include other doubles facts: 0+0, 1+1, 2+2, 3+3.

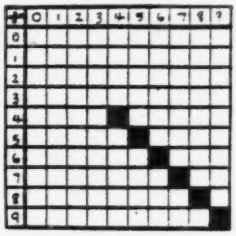
Older learners may enjoy the challenge of mentally adding 2-digit doubles: 11+11, 12+12, 14+14, 32+32, ...

[redacted]

FOCUS ON the following facts:

4+4, 5+5, 6+6, 7+7, 8+8, 9+9.

... and other facts previously studied.



Magic 9s

Verbal Prompt: "See a 9? The pattern will help."

There are 10 Magic 9s to be emphasized: 9 + (4,5,6,7,8) and their turnarounds.

[redacted]

For beginners, check for firm mastery of addition Counts Ons, Zeros, Doubles, Easy 10-Frame facts--and related subtraction facts. *With older students*, review the addition concept before presenting addition 9s.

[redacted]

Note: When addition 9s are taught as suggested below, most second semester first grade (and older) students can recognize and use the pattern effectively for learning them. Should beginners persist in counting vs using mental strategies for +9s, provide alternate assignments. There is no need to introduce finger counting. In another year these children will be more mature and developmentally more "ready" to use number patterns for learning +9s.

Grade 2 level students and beyond: 9s typically would be included among the "easy" addition facts. They in fact may be treated among the first addition facts studied during the year. The challenge of recognizing and using the pattern frequently sparks renewed interest in learning the facts.

---

* *Magic 9s.* (Materials: Calculator [overhead calculator, if available]; individual 10-frames [p. 154]; small counters.) Write three "Magic 9s" facts on the board: (e.g. 9+6, 4+9, 9+7). For each, have one student use a calculator to find the sum. All others should check using counters or 10-frames. Record answers on the board. When complete, study all three facts. "Is there a pattern to what the numbers do?" (Many children will recognize that: (1) each answer begins with 1 ten; and (2) the one's digit in each sum is one <u>less</u> than the number being added to 9. Use counters to

"Magic 9s."

* *Beat the Calculator!* (Materials: Calculator, individual chalkboards; flash cards for Magic 9s.) As you flash each card, students try to write and show the Magic 9 answer before a student on the calculator says the answer. Frequently call on a student to tell the "trick" for getting the answer so fast. (Providing opportunity for this last step is important. Many students need to verbalize an idea in order to make better sense of and retain it. From time to time, let students use counters to check that correct answers are given when the pattern or "trick" is used.)

* *"Hot" Number.* (Materials: Small note paper.) After students write one Magic 9 on their paper say, "The*hot* number is 13 ." Students holding 9+4 or 4+9 stand, show it to you, then quickly trade facts with someone who is seated. As you continue for other 9s, occasionally call on a student to tell the pattern or "trick" for Magic 9s.

* *Thumbs Up!* (Materials: Flash cards--9s and other known facts.) As you flash the cards students hold thumbs up and answer just the Magic 9s. Otherwise, it's thumbs down. Occasionally call on a student to tell the Magic 9s pattern or "trick."

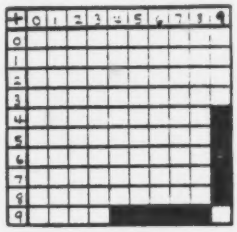
---

* *Pass!* (Materials: Small note paper; flash cards for 9s and other known facts having answers 11-17.) As you "number off," 11-17, students write the number they say on paper. Then, as you show a flash card, all students with the answer stand, show you their number, and quickly trade cards with someone who is seated.

FOCUS ON the following facts and their turnarounds:

9 + (4,5,6,7,8)

...and other facts previously studied.



Doubles + 1

Verbal Prompt: "Think doubles to help."

There are 6 Doubles + 1 facts: 5+6, 6+7, 7+8, and their turnarounds.

[redacted]

Firm mastery of related doubles is necessary.

[redacted]

(**Materials** for the following three activities: Doubles cards [p. 161] and Doubles +1 cards [p. 167]).

* *Match to the Double.* Display the doubles facts aside their pictures along a chalk ledge or on a bulletin board. Distribute the **Doubles + 1 pictures** and ask students to **place each by its "helper" Double.**

* *Find the Helper.* Tape each **Double +1 fact** and its picture back to back. Then distribute these to students, fact side up, and ask them to **place each by the "helper" Double.** When checking, encourage students themselves to say (for example):

"6+6= 12, so 6 + 7 is 1 more (13)."

* *Fact Match.* As in *Find the Helper*, but this time remove the Doubles pictures from the display, so that students are required to **match written facts--a Double +1 to a Double**, and tell the answer for both.

---

* *One Different.* (Materials: Flash cards: Doubles +1 mixed with other known facts.) Remind students Double +1 addends are just "one different" from each other. Flash the cards and ask students to hold thumbs up for "one different"/Double +1 facts, otherwise thumbs down. "Good! Thumbs up for 6 + 7! The helper double is..." (6+6)"...and that's 12, so 6+7=..."

* *One and Only.* (Materials: One and Only cards for Doubles and Doubles +1 cards [p. 171].) Students can play *One and Only* using Old Maid card rules, or leave out the One and Only card and play *Concentration* (common card game rules).

* *Doubles + 1*, Funfax Addition/Multiplication/Division Microcomputer Program, Rich et al. (1985).

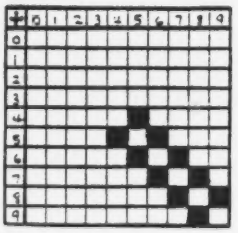
---

* *Five Up.* (Materials: Flash cards for Doubles +1 and other known facts.) Start a whole class drill with flash cards for five easy facts. After a round or two , replace an easy fact with a Double +1. Continue, making sure the Double +1 "comes up" more frequently than others. As students become more confident of this Double +1, replace a second easy fact with a different Double +1, and continue.

FOCUS ON the following facts and their turnarounds:

5+6, 6+7, 7+8,

...and other facts previously studied.



"Make 10, Add Extra" Facts

Verbal Prompt: "Make 10, add extra."

There are 10 "Make 10, Add Extra" facts: 7+4, 7+5, 8+4, 8+5, 8+6
and their turnarounds.

[redacted]

Children should:

1. Have strong visual imagery for the 10-frame. See:
   * *10-Frame Flash.* (Time and Again Activity 8, p. 23)
   * *Frame Fill.* (Time and Again Activity 9, p. 23)
2. Have firm mastery of 10 sums.
3. Mentally be able to add 10 to a 1-digit number.
   Examples: 10+4, 10 +6 ...

[redacted]

* *Part of It is 10.* (To be written. Use 10-frames, p. 154, to "Make 10, Add Extra.")
* *Write it, Then Check It.* (To be written. Write the answer, then use 10-frame to check.)

---

* *Tell What You Think.* (To be written. Write the new facts of this lesson on the chalkboard with a 'think' bubble by each. Invite students to tell what helps them work out answers fast when they don't know an answer. Some students may: (1) Use +9s they know to answer +8s ("8 + 5? Just 1 less than 9 + 5, which I know.") (2) Use a known double ("6 + 8? That's 6+ 6 and 2 more."; **or** "take one from 8, put it on 6. That's 7 + 7.") **or** ...

* *Will 10 Help?* (To be written.)

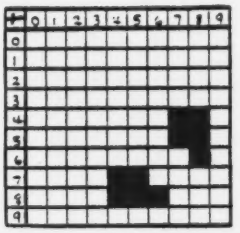
---

* *How Many?* (Materials: Flash cards for the "Make 10, Add Extra" facts and other known facts.) Flash the cards and see how many the class can answer in a minute. Repeat a second time and see if this record can be broken.

FOCUS ON the following facts and their turnarounds:

7+4, 7+5, 8+4, 8+5, 8+6

...and other facts previously studied.



Count Backs

Verbal Prompt: "If the number you take is 3 or less, count back."

There are 27 Count Backs: (12,11,10,9,8,7,6,5,4,3,2) - 1;
(11,10,9,8,7,6,5,4,3) - 2;
(10,9,8,7,6,5,4) - 3

[redacted]

Firm mastery of related addition facts and adequate counting skills are important prerequisites.

* *How Many Numbers Do You Hear?*
* *Say and Stand.*
* *Touch Your Toes.*
(Time and Again Activities 5-7, pp. 21-22)

* *Say and Peek!* (Materials: Demonstration clock.) Point to one number on the rim of the clock while shielding those immediately before that number with your arm or a paper. Direct students to count back two or three numbers, beginning with the number shown. Uncover and check.

Note: When counting back two or three numbers: vocally emphasize the first number, pause, continue counting, and emphasize the last number said.

Example: **11**, ... 10, 9, **8**.

[redacted]

* *Break Off!* (Materials: 12 unifix cubes for each pair of students). Let students team with a partner and take turns "working the cubes". Write a Count Back fact on the board (e.g. 9-3). Direct students to make trains of nine, say "9" (and pause) as they hold the train with both hands, then count backwards to their partner as, one by one, they break off three cubes. Circulate to check for accuracy in this count. Check by modeling the correct count yourself or calling on a child to show what they did. Repeat for other Count Backs.

[redacted]

* *See and Count Back.* (Materials: Flash cards for Count Backs). As you show each card, say the minuend. Children should count back aloud to derive the answer.

* *Choose!* (Materials: Flash cards for Count Backs mixed with other subtraction facts.) As you flash the cards have students hold "thumbs up" every time they see a count back. Each time, call on a child or (occasionally) you yourself count back aloud and give the answer.

When students can comfortably count back to find answers, do:

[redacted]

* *Break a Stick* and **Write Fast!*
(Time and Again activities 11 and 13, p. 25)

* *Number Fact Wheel.* Games and Directions for Parents, p.133 . Adapt to -2/+2 and -3/+3. On the front of each wheel write: "Don't know? Count Back. "

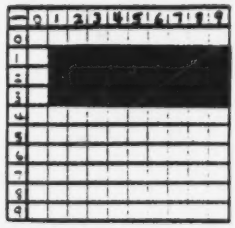
---

* *Winners Up!* (To be written.)

[redacted]

FOCUS ON the following facts:

10-1, 9-1, 8-1, ...2-1; 11-2, 10-2, 9-2, ...3-2; 12-3, 11-3, 10-3,...4-3.
...and other facts previously studied.



When students feel confident with Count Backs, provide "add to check" activities, find "partners," and make families. Adapt the activities of pp. 64-66.

Zeros

"Take all" or "Take None"

There are 19 Zeros: (9,8,7,6,5,4,3,2,1,0) - 0; 9-9, 8-8, 7-7, 6-6, 5-5, 4-4, 3-3, 2-2, 1-1.

These facts tend to be very easy for children. A firm mastery of the subtraction concept and of related addition facts are adequate prerequisites. The concept of subtraction is reinforced in the "Teach Strategy" activities.

---

* *You Take.* Model "taking none" as well as "taking all" the counters. (To be written.)

---

* *Stand Up!* (Materials: Flash cards for Zeros and other subtraction facts previously studied; individual chalkboards). Flash the cards, one at a time. Students stand up when they see a Zero fact. (Call on one student for the answer.) They otherwise write the answer on their boards and show it.

---

* *Read My Mind!* (Materials: Flash cards for Zeros and other subtraction facts previously studied; individual chalkboards.) Draw a card and place it between the pages of a book so the numbers cannot be seen. Tell the answer and ask students to try to "read your mind" by guessing the hidden fact. Direct children to write their guess (with answer) on their boards and show it. All with correct answers get tally point; all "mindreaders" guessing the hidden fact get 2 points. If you like, play boys vs girls, rows or groups vs each other.

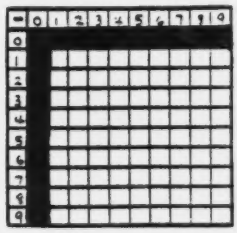
* *Touchdown!* (Materials: Flash cards for Zeros and other subtraction facts previously studied; individual chalkboards.) Appoint one child as referee. Flash each card then let it fall gently to the desk. When it touches the desk the referee calls "Touch!" and the teacher says the answer. Students who write a correct answer before the referee makes a call earn one tally point.

[redacted]

FOCUS ON the following facts:

(9,8,7,6,5,4,3,2,1,0) - 0; 9-9, 8-8, 7-7, 6-6, 5-5, 4-4, 3-3, 2-2, 1-1

...and other facts previously studied.



When students feel confident with Zeros, provide "add to check" activities, find "partners," and make families. Adapt the activities of pp. 64-69.

Subtraction Doubles

Verbal Prompt: "Picture the double."

There are 6 Doubles: 8-4, 10-5, 12-6, 14-7, 16-8, 18-9.

[redacted] Firm mastery of addition doubles is necessary.

[redacted] * *Can You See It?* (To be written. Use the Doubles hide-and-seek cards, p. 176. For directions in using these cards, see p. 150.)

[redacted] * *Write, Show and Tell!* (Materials: Flash cards for Doubles and other subtraction facts previously studied; individual chalkboards). Students write the answer to each card flashed on their boards and show it. The teacher then calls on a child who should describe it (e.g. " a Count Back" or "a Double.") IF THE FACT IS A DOUBLE, the student who answered can call on a classmate to name the Doubles picture.

* *You Show Me!* (Materials: Flash cards for Doubles and other subtraction facts previously studied; two small pieces of paper for each child). Have students write "Count Back" on one paper scrap and "Double" on the other. Children place one paper in each hand and, for each card flashed, hold up "Count Back" or "Double" to identify the fact. In response, say: 1) "Good, a Count Back! --11, ...10, 9, (pause so children can say the answer); OR 2) "Yes, 14-7 is the calendar Double, 14-7= (pause so children can say the answer).

[redacted] * *Seven is Heaven.* See p. 135, Games and Directions for Parents. Include Count Backs and Doubles.

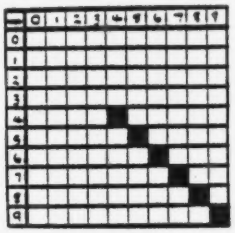
* *Catch a Leaf/Snow Ball/Snowflakes/Christmas Gift.* (Materials: Simple seasonal pictures cut from construction paper, one per student. Write a subtraction Count Back or Double on each picture.) Name the activity to fit the picture. Write a number (1-9) on the board. If this number is the answer to the fact they have in hand, students stand. Then, before you count to five, these students trade facts with someone who is sitting. Repeat.

[redacted]

FOCUS ON the following facts:

8-4, 10-5, 12-6, 14-7, 16-8, 18-9,

...and other facts previously studied.



When students feel confident with the subtraction Doubles, use the hide-and-seek cards to "add to check.", find "partners," and make families.

Adapt the activities of pp. 64-69.

Also do: * *Write Fast!* (Time and Again Activity 13, p. 25)
* *Join Your Family.* (Time and Again Activity 16, p. 27)

Easy 10-Frame Facts

Verbal Prompt: "See 10- ? Think of the 10-frame to help."

There are 7 new Easy 10-Frame Facts: 10-4, 10-6, 10-7, 10-8, 10-9; also: 9-5, 9-4.

Review any other Easy 10-Frame facts not previously mastered.

[redacted]

1. Check for strong visual imagery for the 10-frame. Refer to:

   * *10-Frame Flash.* (Time and Again Activity 8, p. 23)
   * *Frame and Fill.* (Time and Again Activity 9, p. 23)

2. Check for firm mastery of related 10 sums.

   * *Make 10.* (Materials: Individual chalkboards; 10-frame hide-and-seek cards [p. 184] or10-frame on overhead.) Say or write a number (2-9) on the classroom chalkboard. Have students write on their individual boards the number which, when added to your number, makes 10. Call on individual students to check selected problems using hide-and-seek cards (same basic procedure as on p. 150) or an overhead 10-frame.

[redacted]

* *Put In, Take Away.* (Materials: A 10-frame and ten countersfor each pair of students; flash cards for the Easy 10-Frame facts.) Show a fact card and have students take turns modeling the subtraction on their 10-frames: One child puts in 9 or 10 counters; the second child takes away part of the counters as to model the fact shown. Children take turns writing the fact (and answer).

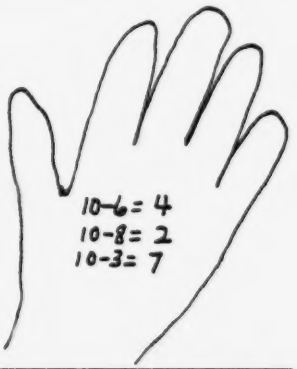
* *Say and Check.* (Materials: A 10-frame and ten counters for each pair of students; individual chalkboards; flash cards for the Easy 10-Frame facts.) Students write the answer to each card flashed. From time to time, when the teacher calls "Check!" students work together with a partner to check their answer on a 10-frame. (Alternately, use an overhead 10-frame or the10-frame hide-and-seek cards to check.)

[redacted]

* *Yes or No.* (To be written.)

[redacted]

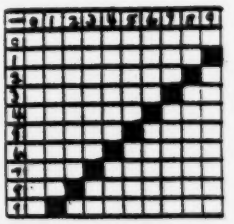
* *Cover!* (Materials: Scrap paper and masking tape.) Have students trace their hand and write three Easy 10-Frame facts with answers on it. Tape it to their desk top. As you pass by a child during the day, whisper "Cover!" and say one fact. If the child answers correctly, draw a star on the ring finger.



[redacted]

FOCUS ON 10-4, 10-6, 10-8, 10-9,...

and any other Easy 10-Frame facts previously studied.



When students feel confident with the Easy 10-Frame facts, provide "Add to Check", Partner and Family activities as on pages 64-69.

Add to Check

Clearly, the most efficient way to answer UNKNOWN subtraction facts is to use known addition facts to provide the answer.

Example: I know 10-6=4 because 6+4=10.

Very young and developmentally immature children find it difficult to reverse their thinking in abstract situations. They tend to find it difficult to "think addition" when the problem in front of them is subtraction. Indeed, for these children, there may be a time lag between recognizing the subtraction-addition relationship and using it to provide answers to unknown subtraction facts. This belief has prompted the careful cultivation in this resource of efficient counting and picture strategies to help answer "easy" subtraction facts.

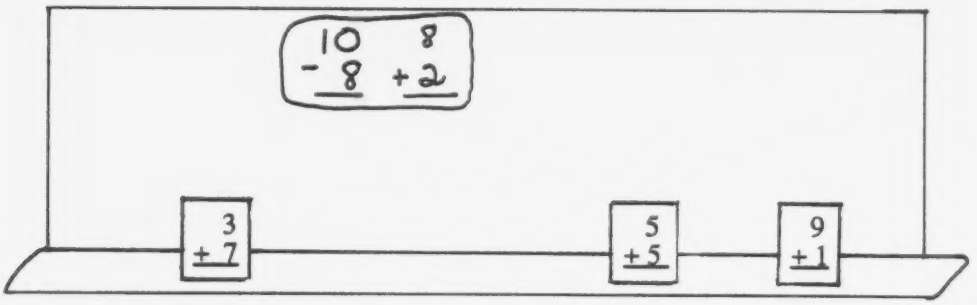
A simple way gradually to build the subtraction-addition relationship and its use is through "Add to Check" activities such as those presented below.

1. * *Break a Stick.* (Time and Again Activity 11, p. 25)
2. * *Apart and Back Again.* (Materials: For each pair of students, up to 18 kernels of popcorn or other counters and an individual chalkboard.) Let students work in pairs. One child writes a subtraction fact (with answer) on the chalkboard. The other uses counters in two hands to show first the subtraction, then the "add to check" fact. The first child writes the add to check fact as it is illustrated.

Example:

	Child 1	**Child 2**
Subtract:	10 - 6 4	Starts with 10 kernels of popcorn. Moves 6 kernels away and says: "10-6=4."
Add to check:	10 6 - 6 + 4 4 10	Moves the 6 kernels back and says: "**Because** 6+4=10."

3. * *Find It, Write It.* Coming in from lunch or gym, or ..., children pick up subtraction fact from a box near the door and try to locate the "Add to Check" fact along the chalk ledge.



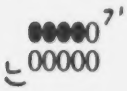
Children copy the subtraction and its "Add to Check" fact onto the chalkboard, then draw a box around it. Students return the flash cards to the flash card boxes on the teacher's desk (addition in one box, subtraction in another).

They then return to their seats, copy and complete as many of the problems written on the board as is possible in the time you allow.

When you decide time is up, get students to tell you what answers to write for the board problems. Frequently comment: "Yes, 10-3 is 7 <u>because</u> 3+7=10." Collect papers, if you like, after students have checked their work in this way.

4. * *Write Fast!* (Time and Again Activity 13, p. 25)

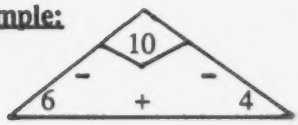
5. * *Pair Up.* (Materials: Page of subtraction facts.) Suggest that students circle three favorite problems on the page and do these first. Beside each, they should write the "Add to Check" fact. Provide paper on which they can draw a picture of one pair of facts.

10-6=4 because 6+4=10 

OR, for "10-" Facts, let students use the 10-Frame Hide-and-SeekCards. Invite students to manipulate the cards to show that 10-6=4 because 6+4=10.

6. * *Two Will Do* . Use Triangle Cards, p. 214, to write a subtraction fact and its "Add to Check" fact.

Example:



10-6=4 because 6+4=10
10-4=6 because 4+6=10

7. * *Bottom Up* . Suggest that one quick check is to place a finger over the minuend of the completed problem and add the other two numbers. Then lift the finger to check that the total is the minuend.

8. * *On the Back.* (Materials: Addition/Subtraction flashcards, p. 198.) Show a subtraction card and ask students to tell you the Add to Check fact on the back. Show the flip side of the card to check.

Subtraction Partner Facts

It will prove helpful to students in the long run if you take the time to focus on related subtraction facts. These are facts that have the same *whole* and the same *parts*. They make good **partners.**

For example: 10-7=3 and 10-3=7; 10-2=8 and 10-8=2.

Activities to Build the "Partner" Relationship for Subtraction Facts

1. * *Break Up*. (Materials: 10 unifix cubes; individual chalkboards.) Gather students around you. Ask students to write the subtraction problem you dictate on their slates (e.g. 10-7).

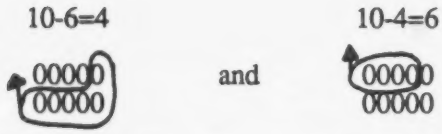
   Make a unifix train on the table or floor to present the "whole" (10). Ask students to write the number that will be left when you break off 7. (3. Break off the seven and, if necessary count to check.)

   Now show the 10 and break off 3. Without erasing, have students write a second fact to match this action. (10-3=7) "Yes, 10-7=3 and 10-3=7. These two facts have the same*whole* (10). Circle the 10 in each problem on your slate. These two facts have the same *parts* (7 and 3). They make good **partners.**" Repeat with other facts.

2. * *Find Your Partner*. (Time and Again Activity 15, p. 26)

3. * *Find It, Write It.* Adapt Add to Check Activity 3, p. 65. Have students find and write partner facts.

4. * *Side by Side*. (Materials: Workpages of subtraction facts.) Suggest that students circle three favorite problems on a page and do those first. Beside each, they should write the partner fact. Provide paper on which they can draw a picture of <u>one pair</u> of facts:

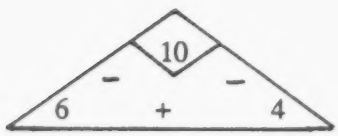


Same whole, same parts: Partner Facts

Better yet, provide small counters and move from desk to desk as students use counters to illustrate a pair of partners facts.

5. * *Two Will Do* . Use Triangle Cards, p. 214, to write a subtraction fact and its related partner fact.

Example:



10-6=4 and 10-4=6

Families of Subtraction Facts

(To be expanded for the final draft.)

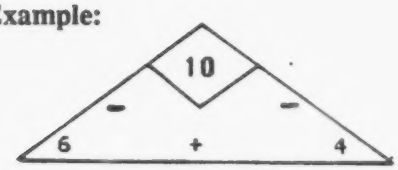
Sample facts: 10-6=4, 10-4=6, 4+6=10, 6+4=10

1. * *To Know One is To Know All.* Use chips in two hands, apart/together, to illustrate all the facts in one family.

2. * *Join Your Family.* (Time and Again Activity 16, p. 27)

3. * *Write It All.* Use Triangle Cards, p. 214, to write all the facts in a given family.

Example:



10-6=4, 6+4=10

10-4=6, 4+6=10

4. * *Tell What's Missing.* Draw one facts from a can or deck. Children should write on their slates others in the same family.

Count Ups

Verbal Prompt: "When the numbers are 'close neighbors', count up."

There are 15 Count Ups. These are facts in which the numbers are "Close Neighbors"--(12,11) - 9; (11,9) - 8; (9, 8) -7; (9,8,7) - 6; (8,7,6) - 5; (7,6,5) - 4. In each case the *answer* is 1, 2 or 3.

NOTE: If beginners deal with Count Backs, Zeros, Doubles, and Easy 10-Frame facts in the manner suggested on pp.56-69, then some students won't need to count to work out answers to the new facts of this lesson.. They'll be able to use addition or subtraction partner facts to answer the new facts.

Examples: "9-7--that's 2, because 7+2=9." **OR**
"9-6--that's 3, because 9-3=6."

Try to identify students who can handle mental relationships like these (e.g. "addition give the subtraction answer", and nurture their thinking in the "concrete" through the pp. 64-66 activities. For those students who developmentally are not "ready" for anything but counting, try the suggestions which follow.

[redacted] Check for mastery of the **Count On** and **Auditory Patterning** activities outlined on pp. 18,19, and 21. Firm mastery of related addition facts and all other "easy" subtraction facts (i.e. Count Backs, Zeros, Doubles, and Easy 10-Frame facts) is important.

[redacted]

* *Touch the Part You Know*. (To be written. Using counters or dots on individual chalkboards, children touch the number of counters (circle the number of dots) taken, and count up to the "whole."

[redacted]

* *Count Up!* (Materials: Count Up cards [Thompson, 1986] p. 189.) Show each card, say the known part (e.g. "8" in 11-8), and pause so students can count up and tell the answer. If necessary, especially in the beginning, this count could be "out loud."

* *Thumbs Up!* (Materials: Flash cards with a mix of Count Ups and other known facts. Students hold thumbs up for Count Ups, thumbs down otherwise. After each Count Up is identified, say the known part and pause so students can count up and tell the answer. Other facts should not be answered. Watch that students don't count for facts they already know.

* *Simon Says.* (Materials: Individual chalkboards.) Give directions like: "Simon says, write a Count Up." "Simon Says, write a Double." Facts and their answers should be written on the boards each time and shown. Occasionally leave out the phrase "Simon says," upon which students should do nothing (Simon didn't say!).

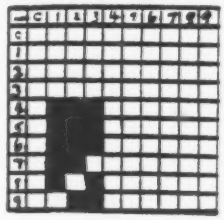
Note: At some point tell students: "If you count quietly (to yourself) and quickly, you'll say the answers so fast everyone will think you just 'know' these facts."

[redacted]

* *Race Row/Box* . (Materials: Worksheet with rows of Count Ups and other easy facts.) Give students time to "study" the first two rows on the page. Help them with any "hard" facts. Then say, "Your *Race Row* is row___" (choose 1 or 2). Allow about 2 seconds/ fact, then call "Stop!" and check answers. Let students draw stars by rows if they beat the time and had correct answers. Repeat for other rows/boxes on the page.

[redacted]

FOCUS ON: (12,11) - 9; (11-9) - 8; (9-8) - 7;
(9,8,7) - 6; (8,7,6) - 5; (7,6,5) - 4
...and other facts previously studied.



When students feel confident with Count Ups, provide Add to Check, Partner, and Family activities as on pp. 64-69.

Other Teen Minuend Facts

There are 26 Other Teen Minuend Facts: 11 - (7,6,5,4); 12 - (8,7,5,4); 13 - (9,8,7,6,5,4); 14 - (9,8,6,5); 15 - (9,8,7,6); 16 - (9,8,7); 17 - (9,8).

Firm mastery of related addition facts is important. If they are not memorized, the most efficient way to derive answers to the teen minuend facts of this lesson is to use the related addition facts to help. This basic approach and alternatives are discussed below and in the Zero Finger lesson of p. 76.

1. **Basic Approach**: Use addition to answer *UNKNOWN* subtraction facts.

   Example: 13-5--that 8, because 5+8=13."

   Verbal prompt: "Addition gives the subtraction answer."

   Step 1: Focus on the "Addition gives the subtraction answer" idea.

   * *Break a Stick* and *Write Fast!* (Time and Again Activities 11 and 13, p. 25.)

   * *What's on the Back?* (Materials: Subtraction cards for teen minuends --related addition fact on the back of each; individual chalkboards.) When you show the subtraction side of each card, students should write the "add to check" fact on their boards and show. Show the flip side of the card to check.

2. **Alternatives to using addition**. With special facts, some students find the following ideas helpful:

- Use number patterns for "-9" facts.

  * *Easy 9s.* (To be written to emphasize that the answer digit is one more than the one's digit of the minuend.) Other number patterns which some students or teachers may like include:

a. The sum of the minuend digit is the answer.
   Example: 14 1+4=5
   -9

b. Subtract 10 and add one.
   Example: 14-10=4, so 14-9=1 more (5).
   And, for the related subtraction facts, the "partner" facts:

c. When the sum of the minuend digits = the part taken, the answer is always 9.
   Example: 16 1+ 6=7, so the answer = 9.
   -7

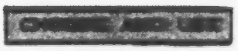
Note: At least during initial teaching, most teachers find it best to select from those strategies rather than selecting all.

- Use the "one more than" idea for facts with minuends near 10 and facts near a Double.

  Examples: 10-6=4, so 11-6=1 more (5).
  14-7=7, so 15-7=1 more (8).

  * *One More.* (To be written.)

- If none of the above seem to help, and students are in late Grade 2 or beyond, try the Zero Finger lesson on p. 76.



* *Snap!* **For use with the "use addition to help" strategy.** (Materials: Subtraction flash cards; individual chalkboards.) As you flash the cards students should write and show each answer. Say the answer aloud when

* *Snap!* **For use with the "use addition to help" strategy.** (Materials: Subtraction flash cards; individual chalkboards.) As you flash the cards students should write and show each answer. Say the answer aloud when most students have shown it. From time to time snap after you say an answer. Tell students this is the cue to write the addition fact that helps with the answer. (For 13-7, for example, accept either 7+6=13 or 6+7=13.)

* *SNAP!* **For use with the "Pattern 9" or "One more than" strategy.** (Materials: Subtraction flash cards.) Write on the board the target group for today's drill: "Pattern 9s" OR "One more than a 10" OR "One more than a double." Then, as you flash the cards, students should snap whenever they see a fact from the targeted group. Call on one student to answer the fact.

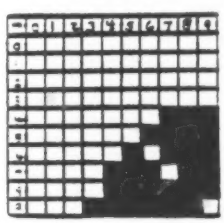
---

* *Answer If.* (Materials: Subtraction flash cards). Write a number (2-9) on the board. Flash cards and direct students to hold thumbs up if the answer is the number written--otherwise hold thumbs down. Call on a child to tell the answer to each thumbs down fact.

FOCUS ON the following facts:

11 - (7,6,5,4); 12 - (8,7,5,4);
13 - (9,8,7,6,5,4); 14 - (9,8,6,5);
15 - (9,8,7,6); 16 - (9,7);
17 - (9,8);

...and other facts previously studied.



Add to Check

If students do not use addition to answer *UNKNOWN* subtraction facts, take the time to build imagery for and use of the subtraction-addition relationship. A simple way to achieve this goal is to carry out many "Add to Check" activities, as on pp. 64-66.

Partner Facts

It will prove helpful to students in the long run if you take the time also to focus on related subtraction facts as on pp. 67-68.

Families of Facts

To consolidate fact work for known facts and to build or reinforce important number relationships, it will prove helpful to take the time to focus on fact families as on p. 69.

Zero Finger Lesson

Verbal Prompt: "Take from 10, add extras."

There are 26 Zero Finger facts: 11 - (7,6,5,4); 12 - (8,7,5,4);
13 - (9,8,7,6,5,4); 14 - (9,8,6,5);
15 - (9,8,7,6); 16 - (9,7); 17 - (9,8).

[redacted]

* Firm mastery of related addition facts so students can "add to check" and increase the potential of their using addition to derive unknown subtraction facts.
* Ability to subtract any number from 10.
* Ability mentally to add 10 to any 1-digit number.

[redacted]

* *Take from 10.* (Materials: Individual 10-frames [p. 160] and small counters.) Write 13 - 5 (or some other "hard"fact) on the board. Ask students to lay out 13 counters, ten in the frame and extras outside. "Now take five counters from your picture." Discuss the different ways students did this. "It doesn't matter which five you take, eight are left."Record the answer to the board problem. Come to the agreement that since it's easy to subtract from 10, "We'll **take from 10** first, then add the extra to find the total left." Repeat using different facts. The goal in this step is to introduce the vocabulary and idea: "Take from 10, add extras."

* *Look Hard!* (Materials: Hole reinforcers [optional].) Ask students to hold up the forefinger (pointer) finger of their right hand, and to look at the fingernail. "Doesn't it look like a zero? ... Say, yes!" (Many teachers write "0" on the fingernail or put a white hole reinforcer on it.) "This is your ZERO FINGER." We'll use it to turn 'hard' subtractions into 'easy' ones."

* *Easy Does It.* (Materials: Paper and pencil.) Ask students to copy four Teen Minuend facts on their paper. Ask students

Example:

13	16	15	14
- 5	- 7	- 8	- 6

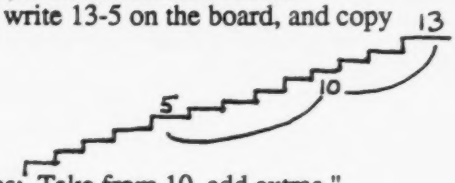
to cover the "3" of the first problem with their zero finger. "We now have a new problem. What is it? (10-5) Which is? (5) and the 3 under your finger make? (13). Get students to repeat, with other facts they have written.

---

* *Your Turn.* (Materials: Individual chalkboards.) Let students team up and take turns writing a "hard" subtraction fact on the chalkboard for their partner to solve. If the fact is unknown, suggest that students might use their zero finger to help. Circulate to check that students are using the procedure correctly.

* *10-Frame Check.* (Materials: 10-frames and small counters.) Get students to write answers then check using the 10-frame. Each time, they should "Take from 10, add extra."

* *The Fast Way Up.* Several days later, **when students become comfortable using the zero finger,** write 13-5 on the board, and copy the drawing. "Using the zero finger is like running up stairs. Instead of starting with the part you know and counting up by ones, you take the whole flight in two steps: Take from 10, add extras."



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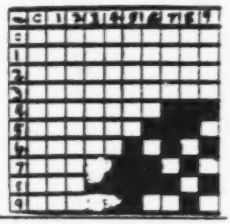
* *Secret Message.* (Materials: Flash cards; individual chalk-boards.) Decide on a message (e.g. "Subtraction is easy.") and write it in code on the chalkboard. Ask students to copy what you write. Then flash each fact and ask students to "write b" (or some other appropriate letter) on the line above your answer."

5 13 1 6 11 12 14 6 8 2 10 8 3 9 12 5 4

Similarly, as you continue, instruct students to write letters above each answer to "crack the code." Library riddle books provide fun ideas for this activity. Pose the riddle and let the math help answer it.

FOCUS ON "hard" teen minuend facts
11 - (7,6,5,4); 12 - (8,7,5,4);
13 - (9,8,7,6,5,4); 14 - (9,8,6,5);
15 - (9,8,7,6); 16 - (9,7); 17 - (9,8)
--any which students do not know.



When students feel confident with the Zero Finger facts, provide Add to Check, Partner and Family activities as on pp. 64-69.

Instructional Strategies To Help Children Memorize Subtraction Facts

NOTE: For each instructional phase outlined below, it is assumed that one would check necessary prerequisites--those given in the Look Ahead activities in the preceding pages. Major prerequisites include firm mastery of related addition facts, concept understanding for both addition and subtraction, and appropriate counting skills.

An overriding instructional thrust in subtraction work should be to lead students to use the most efficient procedure of which they are capable for deriving correct answers to *unknown* facts. "Levels of maturity" in subtraction fact response are outlined below. At each level, a teacher's role would be to nurture growth to a higher level.

Levels of Maturity in Subtraction Fact Response (low to high)

- No attempt or response which indicates inadequate concept understanding.
- Show all : On fingers or with counters, count out the "whole," take some away, and count what's left.)
- Count up/back to determine answer. (Note that these last two levels involve counting.)
- Use an easier, known addition or subtraction fact (mental vs counting strategy)

Overriding goal: mastery of facts to level of automatic recall (memorization), with retention over time. Adequate practice and systematic review after "strategy teaching" is part of a sound basic fact program. On-going review of addition facts should also be carried out.

Instructional Thrusts with Grade 1 Level Students

1. Present Count Backs (Late first or beginning of second semester, after prerequisites are mastered)
   - Teach a way to work out answers fast (see pp. 56-57).
   - THEN carry out "Add to Check" activities (see Time and Again activities 11 and 13, p. 25).

These activities nurture children's understanding of important relationships, and gradually will increase the potential of their using mental vs counting procedures to derive unknown subtraction fact answers.

2. Present Zeros, Doubles, Easy 10-Frame Facts (your choice on which group to do when):

   - For each of these groups in turn: first "teach a way to work out answers fast" (pp. 58-63).

     Note that several of these activities require students to identify facts by strategy for recall. Encourage children to label facts by type: "Count Back," "Double," and so on.

   - When children are fairly comfortable with a strategy provide "Add to Check" activities for that group of facts, as on pp. 64-66.

   - Then carry out "Partner" and "Family" activities as on pp. 67-69.

     These activities further nurture children's understanding of important relationships, and further increase the potential of their using mental vs counting procedures to derive answers to unknown facts. They also consolidate the learning of known facts.

   - Continue systematic review of facts previously studied. (Include addition facts).

3. Present Count Ups. Use the suggestions given on pp. 70-71.

4. Present other subtraction facts ONLY IF students can use non-counting strategies to answer these. For those who cannot, wait until Grade 2.

Instructional Thrusts for Subtraction in Grade 2 and Beyond

Follow the suggestions of pp. 33-34 and those above. Keep these Five Basic Steps in mind:

1. Explicitly intervene to teach correct, efficient ways to work out answers to unknown facts.
2. Later provide "Add to Check" activities
3. Eventually carry out "Partner" activities.
4. Also provide "Family" activities.
5. Continue systematic review.

Teaching Sequences for Multiplication and Division Facts

Overview

(To be expanded slightly in final draft.)

1. Multiplication Concept

2. Easy multiplication facts
   - Any order except zeros and ones not first: (e.g.: 2s, 5s, 9s, 0s, 1s).
   - Intersperse these One a Week Facts:

     "Grade facts": 7x8, 3x4;
     "Floor fact: 8x8;
     Rhyming 6s: 6x4, 6x6, 6x8;
     "Tic Tac Toe fact": 3x3;
     "Three-Week fact": 3x7;
     "Don't be Blue fact": 6x7.

3. Division Concept

4. Easy division facts: those related to easy multiplication facts

   NOTE: To move from x2 to ÷2, from x5 to ÷5 is "too fast." Multiplication facts are not learned well enough to be helpful for answering unknown division facts.

		Strategy
5. Other multiplication facts:	4x (4, 7, 8).....	Double 2s
	8x3, 6x3.....	Double 4x3,3x3
	7x7 (last).....	"The Miner" fact

6. Other division facts: (Use multiplication to help)

General Guidelines:

1. Start early. It's quite all right to start on multiplication and division facts before all addition and subtraction computation work is completed.

   Grade 4 teachers, in particular, will not want to wait until second semester to start multiplication. Besides the summer lag, children otherwise also will have the fall semester lapse between their third grade introduction to multiplication/division and your review of these facts in grade 4.

2. Provide on-going review. Remember to continue drills on addition and subtraction facts previously studied even though your major focus may now be on multiplication and division. Alternate days:
   - addition/subtraction one day,
   - multiplication/division the next...

   Perhaps do an addition or multiplication drill at the beginning of the math period and a subtraction or division drill at the end. Do this even though the major focus of the day is long division, problem solving, measurement, time, or some other topic. Alternately, hold the drills during some other period during the day. It is particularly important that short, daily basic fact drills be scheduled for grade 1-3 students. Grade 4 students typically also need frequent review. Systematic, regular review is basic to most remedial and special education programs involving students who have not mastered all the facts.

3. Find alternatives to skip counting. The picture, clock, pattern and other strategies suggested in this reference help students obtain multiplication answers *faster* than if they skip count by 2s, 5s, 9s, or.... The suggestion is not routinely to encourage skip counting as a method for deriving unknown answers.

4. Consider using the Cross Out sheets:
   - With four or five slowest students in the class. (Create personal Cross Out sheets for reference during homework and tests.) Keep these sheets current: "blacker" each week!
   - With transfer students, to diagnostically determine what they know.

   You might also consider keeping a Class Cross Out sheet on which you mark off facts as they are learned by (most) students during the year.

Picture 2s

Verbal Prompt: "See x2? Think of the picture."

There are 15 Picture 2s facts: 1x2, 2x2,... 9x2--and their turnarounds.

LOOK AHEAD

Mastery of addition doubles is nice. A clear understanding of the multiplication concept is essential.

Because most students know addition doubles, the x2 facts tend to be very easy. Use pictures to help students learn any of these facts they find difficult. Use the following ideas as needed. Visual mnemonics are faster than skip counting. We do not recommend it.

* *Double Trouble*. (To be written. The emphasis is using pictures to derive answers to unknown 2s.)

* *Times 2*, Funfax addition/multiplication/division Microcomputer Program, Rich et al. (1985).

Turnarounds

* *Same Answer, Both Ways*. (To be written.)

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* *What Do You See?* (Materials: Individual chalkboards and flash cards for x2s and their turnarounds.) Order the cards ahead of time so that answers fall into some pattern.

Examples: 6, 16; 8, 18; 2, 12; 4, 14... ;
or 16, 2, 4, 16, 6, 8, ... [where a difficult fact is repeated];
or 18, 2, 16, 4, 14, 6, ... and so on.

As you show and read each card, students should write the answers reasonably small on their chalkboards. They should show their board each time, but should not erase until the end so that an ordered list of answers

can be formed. For any "difficult" fact, reinforce with the verbal prompt (given at the top of preceeding page). At the end of the round, challenge students to identify the pattern in their answers. Reorder cards and do another round. (Alternately, during successive rounds, write facts on the board as you read them.)

* *One and Only!* (Materials: Card set of p. 227.) Let children play Old Maid, Concentration, or Fish-type card games.

SPEED UP

* *Telephone 2s.* (Materials: Pages from an old telephone book; lined or square centimeter paper and pencil.) Let students pair up and take turns selecting any telephone number that doesn't have the digit 0.

x	7	3	8	3	6	5	4
2	14	6	16	6	12	10	8

Both students copy this number onto their paper. When the student who selected the telephone number says "Go!", both students race to multiply each telephone digit by 2 as shown. The winner of each round earns a tally point. Play until teacher calls time.

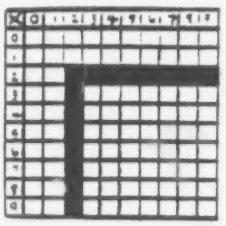
FOCUS ON

FOCUS ON the following facts and their turnarounds:

2x2, 2x3, ... 2x9

... and other facts previously studied.

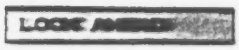
(Continue systematically to review addition and subtraction facts)



Special Facts: One A Week

There are 15 One a Week Special Facts: 3x4, 7x8: "grade" facts; 6x (4, 6, 8): "rhyming facts"; 8x8 "floor fact"; 3x3 "tic tac toe" fact; 3x7 "3-week fact"; 6x7: "Don't be blue" fact--and turnarounds. If students cannot mentally add 16+16 as on p. 94, also include 8x4 (and its turnaround).

It is suggested that the study of each of the above facts be spread out between the study of the "easy" facts identified on p. 30. One might choose, for example, to have a "special fact for the week"-- no matter what other math topics or facts might be focused upon.



A clear understanding of the multiplication concept.

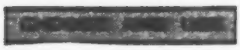
Rhyming Facts

* *They Rhyme!* [6x (4,6,8)] Point out that for the even factors 4, 6, 8--the 6s rhyme: 6 x **4**: **24**; 6 x **6**: **36**; 6 x **8**, **48**. Let students tap out the "beat" of the auditory pattern on their desks.

* *Thumbs Up!* (Materials: Flash cards.) Have students hold thumbs up and answer any rhyming 6s and turnarounds of these facts which they see. Otherwise, it's "thumbs down." Follow through by having students find and do (or count in one minute) all the rhyming 6s and turnarounds of these facts they see on given text pages.

Picture Facts

* *What's the Picture?* **[3x3, 3x7]** (Materials: Old calendar pages which have the first day of the month on a Sunday.) Work separately with 3x3 (the tic tac toe fact) and 3x7 (the three week fact). If students are not familiar with the picture for 3x3, let them play tic tac toe and discuss the 9 positional options for X and O. For 3x7, create a display for calendar weeks: one week, 7 days; two weeks, 14 days; three weeks, 21 days...

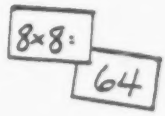


* *Who Has It?* (Materials: Teacher made flash cards in which 3x3 and 3x7 facts appear quite often among other known facts.) Distribute the cards to students. Say, "Who has a... Clock 5?" (... Pattern 9 ...Rhyming fact? ...the Tic Tac Toe fact? the 3-week fact?...) Each time, students who have a fact requested should stand, answer the fact when you call on them, then trade facts with someone who is sitting.

Floor Fact

* *It's On the Floor.* **[8x8]** (Materials: Teacher made cards for "8x8" and "64": several cards for each, answer and fact.) Approach a student with an "8x8=" and a "64" card in hand. Ask the student to read the cards. As the student says "64," let this card drop to the floor. Say, "Pick it up, it's on the floor." Then ask the student to write fact and answer on the board, and direct other students to copy it. Repeat until students sense the rhyme in what is said: "8x8 = 64; Pick it up, it's on the floor."

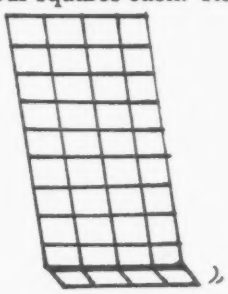
* *Pick It Up!* (Materials: The cards from *It's On the Floor.*) Tape or pin the card sets about the room as shown. As you pass these cards during the next few days, throw the "64" on the floor. As you pass individual desks, periodically ask, "What's 8x8?" Those who give a correct answer should "Pick it up, it's on the floor."



* *Time*: (Materials: Mathematics textbook.) Give students 10 seconds to count the number of "floor facts" on a given page. When time is up, call on individuals to help locate and answer the facts on the page.

The Cut Off Fact

* *4 Less.* **[8x4]** (Materials: Graph paper [p. 233], crayons.) Have students draw and cut out a rectangle having 9 rows of four squares each. Review 9 fours=36, and write 9x4=36 on the board. Ask students to use crayon to mark off 8 rows of four. Show students how to fold and unfold on the crayon line to show 9 fours = 36, so (fold one row under) 8 fours = 4 less (32). Write this new fact on the board beside 9x4=36.



* *Cover!* (Materials: Tape, graph paper rectangle from *4 Less.*) In the center of the rectangle students should write: "9x4 = 36, so 8x4 = 32," and tape it to their desks. As you pass individual desks throughout the next few days, say "Cover! " At this cue the child should put a hand over the facts and tell you what is covered ("9x4=36, so 8x4 = 32.") Occasionally have students illustrate by folding one row under.

Grade Facts

* *Grade Facts.* [3x4, 7x8] and their turnarounds. Say, "You have friends in grades 3 and 4. What grades were they in before that?" (Emphasize order: grades 1 and 2, then grades 3 and 4. Relate to the numbers in: **12=3x4.** Also point out how students complete, *in order*, grades 5 and 6, then grades 7 and 8. Relate to the numbers in: **56=7x8.**)

* *Circle and Do.* (Materials: Worksheet like that shown above and/or mathematics text.) Ask students to circle and do just the grade facts they see on the worksheet. (Focus on only **one** fact in a week if you like.) Follow through by asking students to write answers to the grade facts they circled. Then challenge students to see how many grade facts they can find in one minute on given text pages in their math book.

THE GRADE FACTS (x)

7 X 3	4 X 3	3 X 4	5 X 4	3 X 4	8 X 3

3 x 4 = ___ 4 x 3 = ___ 8 x 3 = ___

4 x 5 = ___ 3 x 4 = ___ 4 x 3 = ___

7 X 8	8 X 3	7 X 7	7 X 3	8 X 8	8 X 7

7 x 8 = ___ 8 x 3 = ___ 8 x 7= ___

6 x 7 = ___ 8 x 9 = ___ 7 x 8 = ___

Some Teachers Create Their Own!
Linda Pauk has shared this idea.

* *Don't be Blue!* (Materials: Small paper and pencil)
"Write 6x7 on your paper. Draw a sad face there if you like, it's a hard fact....Turn the paper over. Write 6x7=42. Draw a happy face on this side because I gave you the answer."

Front

6x7 ☹

Back

6x7=42 ☺

"Now turn the paper to the sad side...6x7--don't be blue. Turn me over, I'm 42!"

* *Hide It!* (Materials: Small paper and tape.) Have students write

the "Don't be Blue" fact and two other hard facts on small paper. (The "Don't be Blue" fact may be the only fact common to all cards, because children will differ in facts that are hard for them.) As you pass a child during the day, take a few seconds to say: "Hide It!" Follow immediately by saying a fact you saw on the card. Frequently, though not always, give "6x7" for the child to answer.

Front	Back
5x8=40 6x7 13-5=8	6x7=42

* *Simon Says*. (Materials: Individual chalkboards.) Give directions like: "Simon says, write the "Don't be blue" fact." "Simon says, write a rhyming '6' fact." Students write and answer facts on their boards, then show their work. Occasionally leave out the phrase, "Simon says," upon which students should do nothing (Simon didn't say!).

* *One and Only!* (Materials: Card set of p. 227.) Let children play Old Maid, Concentration, or Fish-type card games.

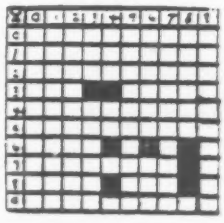
Activities like the following would follow CHOOSE AND USE work for all special facts.

* *Tell Me*. (Materials: Small pieces of note paper; tape.) Ask students to write three facts from this lesson (with answers) on paper and tape it to their desks. As you pass individual desks during the next few days, cover the paper and say: "Tell me,what's...?" (Name one of the facts.) Draw a star beside each fact answered correctly. Provide new paper for different "special" facts as children master the first three.

* *I'm Calling*. (Materials: Flash cards for "special facts" and others previously studied.) Distribute the cards to students and say, "I'm calling for all products in the 40s." (...above 55; ...that are even; and so on). Students holding a fact requested stand, give the answer, and trade cards with someone who is seated.

* *Hot Potato.* (Materials: Bean bag; individual chalkboards.) Let students take turns being catcher for a round of 8 facts. Say a fact then throw the bean bag to the catcher. Give the answer when the bag is caught. Students earn tally marks by writing the correct answer before it is given. Students should show their work each time. Appoint team captains (boys vs girls, row or group vs each other) to keep track of "winners"--those with 6 points per round or better).

FOCUS ON the following facts and their turnarounds:



3x4, 7x8; 6x (4, 6, 8); 8x8;
3x3 ; 3x7; 6x7; 8x4;

... and other facts previously studied.

Clock 5s

Verbal Prompt: "See x5? Think minutes after."

There are 13 Clock 5s: 1x5, 2x5, ... 9x5 -- and their turnarounds.

Ability to tell time on the clock (5-minute intervals *after* the hour).

Reminder: Skip counting involves more steps than using clock times.

We discourage it.

* *The Clock Helps.* Students used to "skip counting" on their fingers will relate well to the clock model if you let them place their fingers on the rim of a small clock and skip count around the clock rim.

* *3-6-9.* (Materials: Individual clocks and flash cards 5x (3,6,9).) One by one, repeatedly show the 3x5, 6x5, and 9x5 cards and have students show and tell the clock time to "match" (15, 30 or 45 minutes after the hour). Provide feedback: ... "Yes, 3 x 5 is 15. We can think of the minute hand on the 3 and 15 minutes after."...

* *Count On or Back.* (Materials: Demonstration clock and individual chalkboards.) Use the clock to demonstrate how 3, 6, and 9 x 5 can help with other facts. For example, since 6 fives are 30, then 5 fives are 5 less (25) and 7 fives are 5 more (35). In a similar manner derive 4 x 5 and 8 x 5. Then, as you move the minute hand on the demonstration clock, have students write (and show you) pairs of related facts on their boards: 3x5=15, 4x5=20; 9x5=45, 8x5=40; 6x5=30, 5x5=25, 7x5=35.

* *What's the Time?* (Materials: Demonstration clock and flash cards for numbers x5.) Distribute the cards to students. As you move the hand to a 3, 6, or 9, students with flash cards "related" to the clock time shown should stand and take turns answering their fact. Reinforce as needed: "Good, 3 fives are 15, so 4 fives are 20 (move hand to show)." When correct, students trade cards with someone who is seated.

* *Times 5* , Funfax addition/multiplication/division Microcomputer Program, Rich et al. (1985).

* *Same Answer, Both Ways*. (To be written. The activity will emphasize that "When you know one fact [e.g. 7x5], you really know two [its turnaround]."

[redacted]

* *SNAP!* (Materials: Flash cards for x2s, x5s and turnarounds) As you flash the cards, students give the answer. If it's a x5 or its turnaround, they snap instead. Immediately call for the answer, then point to the clock (or move the hands of a demonstration model to "match" the fact) and tell the minutes after.

* *One and Only!* (Materials: Card set of p. 227.) Let children play Old Maid, Concentration, or Fish-type card games.

[redacted]

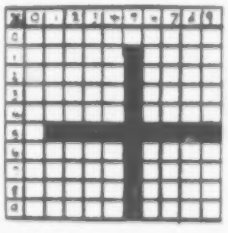
* *Which Is More?* (Materials: Individual chalkboards; flashcards for 5s and other facts previously studied.) Show two cards at a time and have students write, answer and show you the one with the greater product.

[redacted]

FOCUS ON the following facts and their turnarounds:

1x5, 2x5, ... 9x5

... and other facts previously studied.



Ones and Zeros

Verbal Prompts: "x0, always 0."
"x1, stays the same."

There are 36 Ones and Zeros facts: 0x0; 1x0, 2x0, ... 9x0--and their turnarounds.
1x1, 2x1, 3x1,... 9x1--and their turnarounds.

LOOK AHEAD A clear understanding of the multiplication concept.

TEACH STRATEGY **Note:** Instruction for these special facts would emphasize:

1. Students should be able to dramatize/illustrate these.
2. Students should be able to SORT OUT facts which have answers of zero or one from those that do not.

x1 and Turnaround Facts

* *Always the Same.* Extend to problems like 1 x 86; 999 x 1...
* *Equal Answer?* (To be written.)

x0 and Turnaround Facts

* *Easy Zero.* Extend to problems like 39 x 0, 0 x 184...

---

* *Sort!* (Materials: Flash cards) Over a period of several days carry out various types of sort activities:

1. Have students hold thumbs up and answer just the ones which "stay the same". Try including cards from +/-, especially those with ones.

2. Have students form a zero with their fingers if the answer is zero. Try including cards from +/-, especially those with zeros.

3. Have students hold thumbs up, answer just the x1s, and give another x1 problem having a factor greater than 10.

4. As in #3, but for zeros.

5. As in #1 or #2, but let students in some way dramatize or illustrate with counters to check.

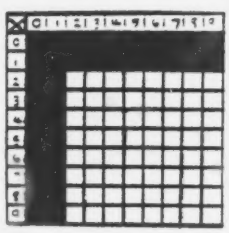
6. Have students work in small groups to sort the cards into piles: those having zero as an answer (e.g. 7 x 0 = 0); or those"which stay the same" (e.g. 6 x 1= 6).

[redacted] * *Go For It!* (To be written.)

[redacted] **FOCUS ON** the following facts and their turnarounds:

0x0, 1x0, 2x0, ... 9x0;
1x1, 2x1, 3x1, ... 9x1

... and other facts previously studied.



Pattern 9s

Verbal Prompt: "See x9? What's the pattern?"

There are 11 Pattern 9s: 1x9, 2x9, ... 9x9 -- and their turnarounds.

[redacted]

Depending on which number and finger patterns are studied, children may need:

1. Finger pattern recognition.
   * *Quick Fingers*. (Time and Again Activity 7, p. 22)
2. The ability to "make 9." That is, given one number (2-9), students can tell a second which, when added to the first, total 9.)
3. The ability to subtract a 1-digit number from a multiple of 10. (Specifically: 20-2, 30-3, 40-4, ... 90-9).

[redacted]

* *Number Patterns*. (To be written.)
* *Finger Patterns*. (To be written.)

[redacted]

* *Thumbs Up!* (Materials: Flash cards--Pattern 9s mixed with other known facts. As you flash the cards, students hold thumbs up for Pattern 9s; thumbs down otherwise. For each Pattern 9 identified, call on a student for the answer. From time to time also call on a second student to tell how it might be worked out if the student didn't know the answer right away.

* *2-Minute Roll/Spin* . (Materials: For each team of two students--one piece of school paper and pencil; 1 die or spinner [numbered 2, 2, 9, 9, 9, 9] paper scraps numbers [2, 3, 4, ...,9]). Determine in advance the

number of rounds to be played. (For 2 rounds, fold paper in half; for 4 rounds, fold paper in 4ths…)

To start, students fold their paper so only one section of the paper shows. Label it "Round 1." At "Go!" one student draws a number and rolls the die. If a "pattern 9" equation can be formed using the numbers, the student writes both the fact and its answer. After 2 minutes the teacher signals "Stop!" See which team has the most x9s (correctly answered). Review the pattern/approach emphasized. Quickly flash the x9 facts (in order) and give answers so students can check their own work. Do other rounds as time allows. At the end, teammates write their names on their paper and turn it in.

* *One and Only!* (Materials: Card set of p. 227.) Let children play Old Maid, Concentration, or Fish-type card games.

SPEED UP

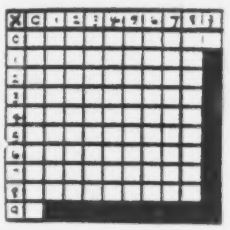
* *Over 45?* (Materials: Flash cards for 9s and other known facts.) As cards are shown students hold thumbs up if the answer is over 45; thumbs down if it's less; and stand up if it's is 45. Then, each time, call on a student to answer: "Good, less than 45, because the answer is…" **or** "Yes, because 4x9 is…"

CHECK UP

FOCUS ON the following facts and their turnarounds:

1x9, 2x9, ... 9x9

... and other facts previously studied.



"Twice As Much" Facts

Verbal Prompt: "An even factor?
Do half, then double it."

There are 9 "Twice As Much" facts: All facts not previously studied: 4 x (4, 6, 7, and possibly 8); 6 x (3, 7); 8 x (3 ,6). We'll call these "twice as much" fact because each is *twice as much* as some easier, known fact.

LOOK AHEAD

Check for firm mastery of x2s, and of the "one a week" facts 3x3, 4 x 3 and 3x7. Also check that students can add two 2-digit numbers MENTALLY (e.g. 12+12, 14+14, 21+21,....) and possibly also numbers (with regrouping) like 16 + 16.

TEACH

* *Rub It* : **Easy Facts to "Unknown" Facts** (Materials: Crayon and paper.) Using a pencil or crayon, have students draw dots: 2 rows of 6. "How many dots in all?" (2 sixes are 12.) Have students fold the paper to cover the dots--and rub **hard** with a finger nail. "When you unfold--how many will you see?" (4 sixes--12 and 12 = 24.) Label as shown:

2 x 6 = 12

so

2 x 6 = 12
4 x 6 = 24

Repeat for other unknown facts which can be derived by doubling easier, known facts. Make a bulletin board of the drawings. Label the display: "Twice as Much." Be sure students know a fact well before you ask them to double it to form another.

* *Doubling:* **"Unknown" Facts to Easy Facts** (Materials: Flash cards, bulletin board drawings from *Rub It.*) Distribute the drawings and have students: 1) Write fact for each drawing (e.g. 4 x 6 = ___); 2) Fold the paper in half and write the easy "helper" (e.g. 2 x 6 = 12); 3) Orally double the easy fact and give the answer to the harder fact. "Many harder facts are twice as much as some fact you already know. Look for one even factor. Then take half and double the easy fact."

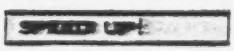
* *Do the Easy Part.* (To be written.)

* *Match and Write.* (Materials: Flash cards for the "twice as much" facts and for their "helpers".) Place flash cards for the "helper" facts along the chalk ledge. Distribute the "twice as much" cards to students. Have them match each to its "easy" helper. At their desks, students should write (and answer) both the helper and the twice as much fact each time a match is made. Summarize by collecting and flashing the "twice as much" facts. Provide reinforcing feedback as needed: ..."Yes, 2 sevens are 14, so 4 sevens are 28."...

---

* *Picture Check.* (Materials: Flash cards; individual chalkboards) Students only write and show answers for "twice as much" facts. From time to time call on a student to identify the bulletin board picture (from *Rub It*) to check.

* *Triple Decker.* (Materials: Flashcards; answers; tape; chalkboard.) Tape each "twice as much " fact on the wall (top edge only), its easy "helper" underneath, and the answer to both facts on the bottom layer. As students go in and out of the room, let them answer, lift and check.

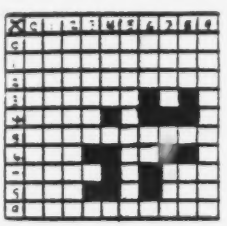


* *Beat the Buzz.* (Materials: Flash cards and timer.) Set the timer and see how many facts can be answered before the timer buzzes. Repeat to beat this record.

FOCUS ON the following facts:

4 x (4, 6, 7, 8);
6 x (3, 7); 8 x (3, 6)

... and other facts previously studied.



Squares and the "Miner"

One Last Fact: 7x7 (the "gold miner fact").

Facts to be reviewed: Other squares: 2x2, 3x3, 4x4, ... 9x9.

LOOK AHEAD

- Clear understanding of the multiplication concept.
- Ask the school's music teacher to teach Clementine ("...Dwelt a miner, 49er...")

---

* *Strike It Rich!* Ask students what they think of when you say "49ers" -- besides a ball team. If they don't respond, say, "The Gold Rush (1849)--during which the miners took a pick (draw a 7 on the board) and a spare (draw another 7). We'll let these 7s be gold picks to remind us: 7 x 7 = 49. This is the "miner" fact.

* *Last Fact.* "One last fact is surely fine, 7 x 7 is 49!." Given that the preceding sequence has been followed, point out to students using a Cross Out sheet that this is the last of the 100 basic multiplication facts to learn.

* *Squares.* (Materials: Graph paper [p. 233].) Ask students to use graph paper to draw a 7 x 7 shape (square). Point out that 7 x 7 (49) is called a square number, or perfect square, from its shape. Ask students to "draw" and label other square numbers. Variation: Project a graph paper grid onto the chalkboard (using overhead projector), and let students use colored chalk to draw and label the squares.These shapes could be traced onto colorful paper for a bulletin board or hall display. Put the 7x7= 49 square on gold paper.

---

* *Pop Up!* (Materials: Multiplication flash cards; individual chalkboards.) Students "pop up" like toast (stand) to answer 7x7 and other squares every time they appear. They otherwise write and show answers written. Let the 7x7 fact "come up" more often than other facts. "Yes, that's the miner fact... 7x7 is surely fine, 7x7 is 49."

* *Square It.* (Materials: Multiplication flash cards with duplicates of "square" facts; individual chalkboards.) As you flash the cards, students write each answer. If the answer is a square, they should draw a square around it. If it's the miner fact, you could allow a phrase from Clementine or have students put two "gold picks" by it.

* *Gold Miner Sheet.* (Materials: Paper and pencil.) Ask studentsto write the gold miner fact at the top of a sheet of paper. Ask them to finger trace over the fact as they say it to themselves. Then have them close their eyes and write the fact in the air, on the chalkboard of their minds. Eyes open, have students fold the top line (pleat the paper) to hide the fact before writing it again. Repeat the folding, writing (and opening to check any time) to the bottom of the sheet. (THIS is the way to write the times tables and spelling words--so that each new line must be recreated rather than mindlessly copied.) At the end, when the paper is unpleated, every line should be "like" the top line. Team captains could count "winners" as they finish.

* *Let's Party!* (Materials: Snack wrapped in gold foil; the pleated sheet from the previous activity.) Trade students a bit of gold (snack in gold foil) for their correctly completed "gold miner" sheet. "Since we now know all the multiplication facts, let's party!" You even have a party song: Clementine!

* *One and Only!* (Materials: Card set of p. 227.) Let children play Old Maid, Concentration, or Fish-type card games.

SPEED UP!

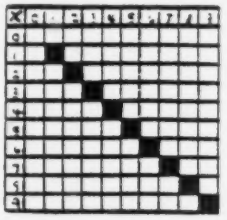
* *Star Shoot.* (To be written.)

CHECK UP!

FOCUS ON the following facts:

7 x 7 and other squares:
(2 x 2, 3 x 3, 4 x 4; ... 9 x 9);

... and other facts previously studied.



Instructional Strategies to Help Children Memorize Division Facts

A. **UNKNOWN "EASY" FACTS -- +2s , +5s, +9s:**

- Prerequisites: Firm mastery of related x facts; ability to tell time on a standard clock to the nearest five minutes.

- FIVE Basic Steps (for each set of facts):

1. **Teach a way to work out answers fast.**
   "See a 2? What's the picture?"

   "See a 5? Think minutes after. Where's the minute hand?"

   "See a 9? What's the pattern?"
2. **Multiply to check.** (When students can work out answers, encourage them to multiply to check. Carry out explicit activities which reinforce the division-multiplication relationship. Follow with written exercises. 35÷ 5 = 7 because 5 x 7 = 35.)
3. **Teach related partner fact.** (For example, when +5 facts are firmly mastered, suggest that students use these to help with unknown related division facts. 35 ÷ 5 = 7, so 35 ÷ 7 = 5. Activities as well as written exercises are needed in this teaching.)
4. **Study fact families.** (Use finger flash cards, "What's On the Back," and other activities and written exercises to consolidate retention of known facts in families like: 5 x 7 = 35; 7 x 5 = 35; 35 ÷ 5 = 7; 35 ÷ 7 = 5.)
5. **Additional drill** in the form of short activities and written exercises will be necessary to assure retention of newly learned division facts in long term memory.

B. **Treat special facts: those with 0 and 1** as a factor. Use small objects to illustrate meanings. Carry out sorts (e.g., those whose answer is 0 (or 1) vs those whose answer is not...).

C. **UNKNOWN "HARDER" FACTS.**

- Prerequisites: Firm mastery of related multiplication facts; previous experiences emphasizing the "multiply to check" (step A-2, above).
- Four Basic Steps (You may want to start with perfect squares: 36 ÷ 6; 64 ÷ 8...):

  For each set of division facts studied:

  1. **Use multiplication facts you know to answer unknown division facts.**
  2. **Teach related partner facts** (Exception: squares).
  3. **Study fact families.** (Squares have "small" families.)
  4. **Additional drill.**

Picture 2s

Verbal Prompt: "See +2? Think of the picture."

There 15 Picture 2s: 4+2, 6+2, 8+2, ... 18+2,
-- and their partner facts: 6+3; 8+4, ... 18+9.

[redacted]

1. A clear understanding of the division concept.
2. Firm mastery of x2s and their turnarounds.

[redacted]

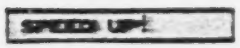
Because most students know multiplication 2s, the +2 facts tend to be very easy. Use pictures to help students learn any of these facts they do not know. Use the following ideas as needed.

* *Do You See It?*
* *Division Pictures.*
* *Divide by 2*, Funfax Addition/Subtraction/Multiplication Microcomputer Programs, Rich et al. (1985).

[redacted]

* *Your Turn.* (Materials: Individual chalkboards.) Direct students to write the even numbers from 4 through 18 at the top of their slates. Students take turns calling out one of these numbers, and others write and answer the +2 fact using the number as the dividend. When students show their work, occasionally provide feedback like: "Yes, 14+2=7. That's the 2-week fact: 14 days, 2 weeks--7 days in each week. Create a

tally system for correct answers, if you like, and set up teams: girls vs boys; rows or groups against each other, and so on.



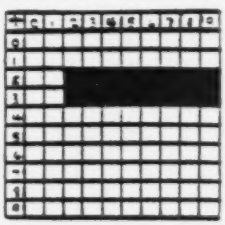
* *On the Draw.* (Materials: Individual chalkboards; flash cards for +2 facts.) Draw a sample stickperson on the chalkboard: head, neck, body, 2 arms, 10 fingers, 2 legs, 2 shoes. As you flash the cards, students try to answer and show their work before you give the quotient. For each correct answer, students draw one part of the stickperson on a corner of their board. See who finishes first!

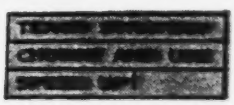
* *Pop Up!* Adapt. p. 47. Distribute cards numbered 2-9.

FOCUS ON the following facts and their partners:

4+2, 6+2, 8+2, ... 18x2,

... and other facts previously studied.





As students become confident of +5 facts, emphasize appropriate "Multiply to check" activities (pp. 104-106). Then provide appropriate Teach, Choose and Use, and Speed Up activities so children learn the related *partner facts* (pp. 107-108). Consolidate by focusing on fact families (p. 109) and providing additional practice.

Multiply to Check

Clearly, the most efficient way to answer UNKNOWN division facts is to use known multiplication facts to provide the answer.

Example: I know 12 ÷ 2 = 6 because 2 x 6 = 12.

Some children cannot use multiplication facts to derive division fact answers simply because the related multiplication facts are not memorized. This is the underlying reason for the suggestion to delay the introduction of a group of division facts until the related multiplication facts are mastered. To move directly from multiplication "2s" to division "2s" typically does not allow adequate time to develop mastery to a point where multiplication facts can be recalled readily to provide division answers or to check them.

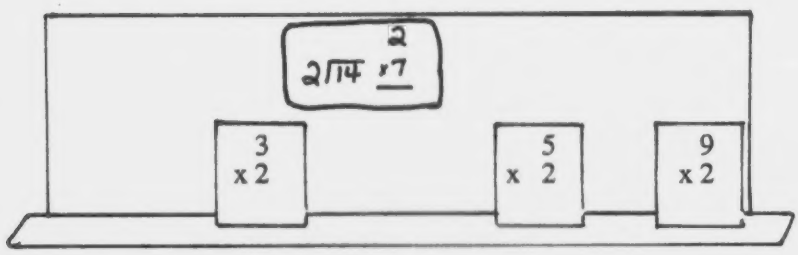
Some developmentally immature students find it difficult to "think multiplication" when given a division problem. Indeed, for these children, there may be a time lag between recognizing the multiplication-division relationship and being able to *use* it to provide answers to unknown division facts. This belief has prompted the careful cultivation in this teaching resource of strategies involving pictures, clock times, and patterns to help answer "easy" division facts. The idea is to allow time gradually to build the multiplication-division relationship and its use is through "Multiply to Check" activities such as those presented below.

1. * *Share!* (Time and Again Activity 12, p. 25.)
2. * *Divy Up--and Take Back Again.* (Materials: unifix cubes, crayon, individual chalkboard and chalk for each pair of students.) Children work in pairs. One child writes a division fact (with answer) on the chalkboard. The other, using unifix cubes forms rows fo equal lengths to represent the dividend (same number of rows as the divisor), and then moves the rows

Example: 12 ÷ 2 = 6

As the first child then pushes the parts together again to illustrate the "multiply to check" step, the second child writes the multiplication fact on the chalkboard (2 x 6=12).

3. * *Find It, Write It.* Coming in from lunch or gym, or ..., children pick up division fact from a box near the door and try to locate the "Multiply to Check" fact along the chalk ledge.



They copy the division and its "Multiply to Check" fact onto the chalkboard, then draw a box around it. Students return the flash cards to the flash card boxes on the teacher's desk (multiplication in one box, division in another). They then return to their seats, copy and complete as many of the problems written on the board as is possible in the time you allow.

When you decide time is up, get students to tell you what answers to write for each board problem. Frequently comment: "Yes, 14 ÷ 2 *is* 7 because 7 x 2 = 14." Collect papers, if you like, after students have checked their work in this way.

4. * *Write Fast!* (Time and Again Activity 13, p. 25)

5. * *Pair Up.* (Materials: Page of division facts.) Suggest that students select three favorite problems on the page and do these first. Beside each, they

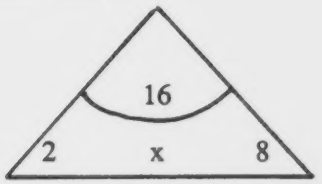
should write the "Multiply to Check" fact. Provide paper on which they can draw a picture of one pair of facts:

$10 \div 2 = 5$ because $5 \times 2 = 10$

OR, for the x2 Facts, let students use the picture cards. Invite students to place a ruler or string on the cards to divide the picture into 2 parts, then remove it as they state the "Multiply to check" fact.

6. * *Two Will Do* . (Use Triangle Cards, p. 214, to write a division fact and its "Multiply to Check" fact.)

Example:



$16 \div 2 = 8$ ***because*** $2 \times 8 = 16$

7. * *What's on the Back?* (Materials: Multiplication/Division flash cards, p. 215, individual chalkboards.) As you show the division side of each card, students write and show the answer and the "Multiply to check" fact. Show the flip side of the card to check. For at least one or two facts, use counters, graph paper or array cards to illustrate the division/multiplication relationship.

8. * *Multiply and Match.* Suggest that one quick check is to place a finger over the dividend of the completed problem and multiply the other two numbers. Then lift the finger to check that the product is the dividend.

Division Partner Facts

It will prove helpful to students in the long run if you take the time to focus on related partner facts. These are facts that have the same *whole* (dividend) and the same *factors* .

Examples: 10 ÷ 2 = 5 and 10 ÷ 5 = 2;
14 ÷ 2 = 7 and 14 ÷ 7 = 2.

Activities to Build the "Partner" Relationship for Division Facts

1. * *Break Up*. (Materials: 18 small counters; individual chalkboards.) Gather students around you and ask them to write the division problem you dictate on their slates (e.g. 12 ÷ 2). Make a long unifix train on the table or floor to present the dividend (12). Ask students to write the number each will get if they share the cubes (6--12 ÷ 2 = 6. Break the train into two equal parts and count to check.)

   Now show the 12 and share among six. Without erasing, have students write a second fact to match this action. (12 ÷ 6 = 2) "Yes, 12 ÷ 2 = 6 and 12 ÷ 6 = 2. These two facts have the same *whole* (12). Circle the 12 in each problem on your slate. These two facts have the same *factors* (2 and 6). **We call them partner facts.**" Repeat with other facts.

2. * *Find Your Partner*. (Time and Again Activity 15, p. 26)

3. * *Find It, Write It*. Adapt Multiply to Check Activity 3, p. 105. Have students locate, write and box partner facts.

4. * *Side by Side*. (Materials: Page of division facts.) Suggest that students circle three favorite problems on the page and do those first. Beside each, they should write the partner fact. Provide paper on which children can draw a picture of <u>one pair</u> of facts:

$12 \div 2 = 6$

000000

000000

and

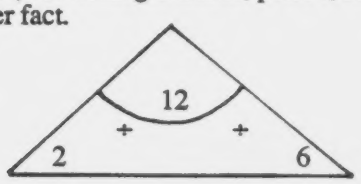
$12 \div 6 = 2$

Same whole, same factors: Partner Facts

Better yet, provide small counters and move from desk to desk so students can use counters to illustrate a pair of partners facts.

5. * *Two Will Do* . (Use Triangle Cards, p. 214, to write a division fact and its related partner fact.

Example:



12 ÷ 2 = 6 and **12 ÷ 6 = 2**

Families of Division Facts

(To be expanded for the final draft.)

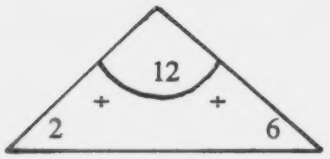
Sample Family of facts: $16 \div 2 = 8$, $2 \times 8 = 16$,
$16 \div 8 = 2$, $8 \times 2 = 16$

1. * *To Know One is To Know All.* (Materials: Chips.) Lay chips in rows to form a rectangular array. Move the rows apart/together, to illustrate all the facts in one family.

2. * *Join Your Family.* (Time and Again Activity 16, p. 27)

3. * *Write It All.* Use Triangle Cards, p. 213, to write all the facts in a given family.

Example:



$12 + 2 = 6$, $2 \times 6 = 12$,
$12 + 6 = 2$, $6 \times 2 = 12$

4. * *Tell What's Missing.* (Materials: Cards with division facts, individual chalkboards.) Draw one fact from a can or deck. Children should write on their slates other facts in the same family.

Clock 5s

Verbal Prompt: "See +5? Think minutes after."

There are 13 Clock 5s: 5+5, 10+5, 15+5, 20+5, ... 45+5, -- and their partner facts: 5+1, 10+2, 15+3, 20+4, … 45+9.

[redacted]

1. Firm mastery of x5s and their turnarounds.
2. Ability to tell time on the clock quickly to the nearest five minutes.

If students can tell time on the clock to the nearest five minutes, the + 5 facts can be very easy. They can use clock times they know to help.

[redacted]

* *The Clock Helps.* (Materials: Demonstration Clock.) Given a Clock 5 in division, students say the "minutes after" and move the hand of a demonstration clock to the answer digit.

[redacted]

* *Divide by 5,* Funfax Addition/Multiplication/Division Microcomputer Programs, Rich et al. (1985).

---

* *Clock It.* (Materials: Clock and individual slate boards for each team of two students; +5 flash cards;) Let students work with a partner. As you show each card, one student sets the clock to match the fact, the other copies, answers and shows the slate. Reinforce frequently by saying: "You see +5--think minutes after. Where's the minute hand?--Yes, on the answer digit." Students trade roles each time.

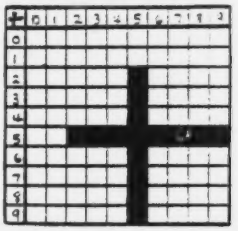
SPEED UP

* *Right On!* (Materials: Flash cards for +5 and other known facts.) As you flash the cards students hold thumbs up if the answer is greater than 5, thumbs down if its less, and stand if it's "right on" 5. When students hold thumbs up/down, call on an individual to tell the answer. "Yes, greater than 5 because the answer is..."

FOCUS ON the following facts and their partners:

5+5, 10+5, 15+5, 20+5, ... 45+5,
5+1, 10+2, 15+3, 20+4, ... 45+9,

... and other facts previously studied.



Multiply to Check

As students become confident of - 5 (facts, emphasize appropriate "multiply to check" activities. (pp. 102-104)

* *What's On the Back?* (Materials: Multiplication/Division flash cards, p. 215, individual chalkboards.) As you show the division side of each card, students write and show the answer and the "Multiply to Check" fact. Show the flip side of the card to check. For at least one or two facts, use counters, graph paper or array cards to illustrate the division/multiplication relationship.

Partners and Families

TEACH STRATEGY
CHOOSE AND USE
SPEED UP

When students have mastered the +5 facts, provide appropriate Teach, Choose and Use, and Speed Up activities so children learn the related *partner facts* (pp. 105-106). Consolidate by focusing on fact families (p. 107) and providing additional practice.

Pattern 9s

Verbal Prompt: "See +9? What's the pattern?"

There are 11 Pattern 9s: 9+9, 18+9, 27+9, … 72+9, 81+9 -- and their partner facts: 9+1, 18+2, 27+3,... 72+8.

[redacted] For those students who use finger patterns for division 9s, immediate finger pattern is important:

* *Quick Fingers.* (Time and Again Activity 10, p. 24)

[redacted]

* *It's in the Numbers!* (To be written, emphasizing number patterns for 9s.)

* *Finger Pattern.* (To be written, emphasizing finger patterns for 9s.)

[redacted]

* *Chain Up.* (Materials: Several construction paper strips for each student--seasonal colors.) Give each student several strips and direct them to write a +9 fact on each. As you pass individual desks during the day, select a strip at random and ask the student to answer it. Give credit both for immediate correct answers and for those correctly derived using a finger or number pattern. Use this opportunity to check that individual students who do not know the 9s are using a correct procedure for deriving them. Those who respond correctly should add their strip to the "class chain" which, when long enough, can be hung as part of the room decoration.

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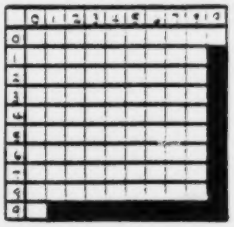
* *Tape Up.* (Materials: Tape rolls for chalkboard; flash cards for +9s and other known + facts.) Before students come into the room write the numbers to nine in order along the board and place several tape rolls under each. As students enter, they take a flash card and tape it beneath its answer written on the board.

FOCUS ON the following facts and their partners:

9+9, 18+9, 27+9, ... 72+9, 81+9,
9+1, 18+2, 27+3, ... 72+8,

...and other facts previously studied.



Multiply to Check

As students become confident of +9 facts, emphasize appropriate "Multiply to check" activities. (pp. 104-107)

Partners and Families

When students have mastered the +9 facts, provide appropriate Teach, Choose and Use, and Speed Up activities so children learn the related *partner facts* (pp. 107-108). Consolidate by focusing on fact families (p. 109) and providing additional practice.

Ones and Zeros

There are 17 Ones and 9 Zeros: 1+1, 2+1, 3+1, ... 9+1;
--and partners: 2+2, 3+3... 9+9.
Also, 0+1, 0+2, 0+3, ... 0+9.

[redacted] A clear understanding of the multiplication concept.

[redacted] **Note:** Instruction for these special facts would emphasize:

1. Students should be able to use counters to dramatize/illustrate these.
2. Students should be able to SORT OUT facts which have answers of zero or one from those that do not. For zeros, they should be able to identify which are "not allowed."

Ones

* *Treats*. Extend to problems like 43+1; 777 + 1...

Zeros

* *All Mine*. Extend to problems like 0+69, 0+298 ...
* *A No! No!* (To be written.)

[redacted]

* *Sort!* (Materials: Flash cards) Over a period of several days systematically mix zeros and ones with other facts and carry out various types of sort activities:

1. Have students hold thumbs up and answer just those +1; having 1 as an answer. Occasionally ask students to "Multiply to check," to give the partner fact, or other facts in the "family."

2. Have students hold thumbs up and answer just those with zeros. Occasionally remind students to "Multiply to check."

3. As in #1 or #2, but let students in some way dramatize or illustrate to check.

4. Let students work in small groups actually to sort the cards into piles: those having 0 as an answer; ...1 has an answer; ... those which are "not allowed;"...

[redacted]

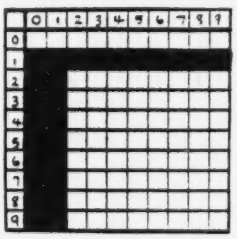
* *Up and Down.* (To be written.)

[redacted]

FOCUS ON the following facts:

1+1, 2+1, 3+1, ... 9+1; and partners: 2+2, 3+3... 9+9;
0+1, 0+2, 0+3, ... 0+9;

...and other facts previously studied.



Take the time to do **multiply to check, partner** and **family activities** as on pp. 104-109. Note that zeros have small families.

Other Division Facts

Verbal Prompt: "Multiplication gives the division answer."

There are 20 other division facts: (**9**,12,18,21,24)÷3; (12,**16**,24,28,32)÷4; (18, 24, **36**, 42, 48)÷6; (21, 28, 42, **49**, 56)÷7; (24, 32, 48, 56, **64**)÷8.

1. A clear understanding of the division concept.
2. Firm mastery of related multiplication facts.

Squares (those in bold, above)

Perhaps the easiest of the remaining division facts are those in which *"the answer is in the problem":* the *squares*. It also is fairly easy with this group of facts to emphasize the "multiply to check" idea.

* *Share*. (Time and Again Activity 12, p. 25)

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* *Read My Mind.* As on p. 58, but use cards just for squares. Remind students: "The answer is in the problem."

* *Stand Up!* As on p. 58, but mix multiplication squares with other known division facts. When needed, remind students that "The answer is in the problem."

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* *Touchdown!* As on p. 59, but mix squares with other facts.

Other 3s (see above)

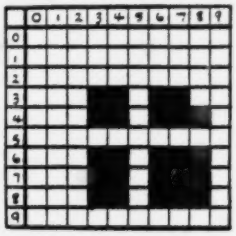
* *Share Out!* (Time and Again Activity p. 12) Emphasize that "multiplication gives the division answer."

* *Multiplication Helps*. (Materials: Unifix cubes.) Write the four new +3 facts on the board. In a separate list, write the related multiplication facts. Have students circle the number in each multiplication problem that gives a division answer. Have students complete the problems, then draw a line between each quotient digit and the corresponding number in the multiplication problem. For at least one problem, repeat the *Share Out* procedure to check.

FOCUS ON the following facts:

(**9**,12,18,21,24) ÷ 3; (12,**16**,24,28,32) ÷ 4;
(18, 24, **36**, 42, 48) ÷ 6; (21, 28, 42, **49**, 56) ÷ 7;
(24, 32, 48, 56, **64**) ÷ 8

... and other easier facts.



When students have mastered the new +3s, provide Teach, Choose and Use, and Speed Up activities so children learn the *partner facts*. Consolidate by focusing on families and providing additional practice. Sample activities are provided on pp.107-109.

Other 4s (see above)

Adapt activities used with +3s. Use multiplication to find the division answer; practice to mastery. Then study partners and families, using the pp. 107-109 activities.

Three Last Facts: 42÷6, 48÷6, 56÷7

Adapt activities used with +3s. Use multiplication to find the division answer; practice to mastery. Then study partners and families, using the pp. 107-109 activities.

Guidelines for "Good" Drill and Practice in the Basic Fact Program

(To be expanded in the final draft.)

Note that, in general, the "best" drill is that which:

1. Comes after, Look Ahead, Teach Strategy, and Choose and Use Activities.
2. Is carried out on a daily basis.
3. Is short.
4. Requires all students to respond.
5. Provides immediate feedback on whether a response is correct.

Vary the drill "routine" to keep students motivated.

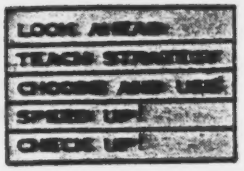
Modifying Activities to Meet Learning Needs of Individual Children

(Learning strengths and weaknesses are considered.)
Examples of each suggestion to be provided in final draft.

I. FOR AUDITORY LEARNERS
- A. Precede all actions and demonstrations with spoken instructions. Each step may comprise two parts:
  1. Oral instructions only.
  2. Oral instructions closely followed by visual stimulus, concrete manipulation or demonstration.
- B. Provide a verbal summary of each step.
- C. Use key words to focus the child's attention, if necessary.
- D. Remove extraneous visual stimuli.

II. FOR VISUAL LEARNERS
- A. Precede all oral instructions by the concrete manipulations,mime demonstration. Each step may comprise two parts:
  1. Presentation of visual stimulus only.
  2. Visual presentation in conjunction with verbalization.
- B. Have children describe mimed or demonstrated action, pictures, concrete manipulation, etc.
- C. Provide a visual summary of each step.
- D. Encourage the child to make mental images of visual stimulus. Provide opportunity for him to reproduce these images (verbal, pictorial).
- E. Use cue cards to focus the child's attention.
- F. Try a silent lesson.

III. FOR KINESTHETIC/TACTILE LEARNERS
- A. Precede all instructions by the physical manipulations of objects by the child. Teacher should guide all manipulations. Each step may comprise two parts:
  1. Physical manipulation only.
  2. Physical manipulation in conjunction with oral instructions.
- B. Remove visual stimulus, if distracting. Have the child close his eyes, or place objects in his hands behind his back.
- C. Provide a summary (physical manipulation of each step).
- D. Use a cueing system to focus the child's attention.
- E. Make use of textured material (pipe cleaners, sandpaper, plasticine), sandtrays and magnetic boards.



Meeting Remedial Needs in Basic Facts

Inadequate fact mastery is a major cause of difficulty in mental math, written computation and estimation. Remedial attempts to alleviate this difficulty might incorporate the following suggestions:

- Start with assessment (see p. 126) and create a cross out sheet (see p. 13) for each operation being focused upon.
- Determine the starting place. Decide upon the sequence in which facts will be grouped for study. See p. 28 and base planning on strategies you feel will be easiest/most helpful for individual students. If possible, delay subtraction/division until related addition/multiplication facts are mastered. In the interim, let students use a cross out sheet for seatwork and tests. Just keep these lists current--cross out a few facts each week as they are learned.
- As you plan, look ahead to "tomorrow" and to "three weeks from now." See p. 16 and start working on skills which are prerequisite to successful learning of strategies you will teach.
- As you plan, build in structured review (over all operations) of facts previously studied.
- Start and end each class period with a quick, motivational review activity. You might need to do addition/subtraction one day, multiplication/division the next, and so on.
- Prior to drill, explicitly teach students strategies such as those given in this booklet for working out answers to unknown facts.
- Help students discriminate when a strategy applies and when it does not.
- Provide verbal prompts such as those given throughout this booklet.

- "Clump" your teaching of strategies. For example, work on doubles and 9s; clock 5s and 9s; count backs and doubles; harder subtraction facts and clock 5s; ... and so on. Each of these combinations involve unrelated strategies, so success with one group of facts is not required before beginning the second.

- Teach "easy" facts first--as quickly as possible (yet insuring mastery)--so you have time to get to facts for which students really need your help. For older students, start with easy but "big" numbers: 9s or doubles or zero finger facts or...

- If possible, communicate with parents and elicit their support. Share terms from the Glossary (p. 142) with them so they better understand what you are doing.

- Provide a carefully structured drill and practice program.

- Systematically integrate problem solving (Time and Again Activity 14, p. 26) and concept review (Time and Again Activity 13, p. 25) with fact work.

- Occasionally focus on subskills which will lead to success in computational estimation and mental math. In particular, help students relate fact work to computing with multiples and powers of ten. For example:

  1. $2 + 6 = 8$, so $20 + 60 = \ldots$
  2. $3 \times 4 = 12$, so $30 \times 40 = \ldots$ Also: If $3 + 6 = 9$, then $23 + 6 = \ldots$

- Consider direct extensions/applications of fact work. *Example based on flash card drill:* Say to students, "I don't want the answer this round--just the product plus 4. Do it in your heads, write and show me just the final answer."

  This skill is needed in the "multiply and add" step of 2-digit multiplication.

Case Studies

(Commentary for providing a remedial program for two students, of different ages and backgrounds, based on profiles provided by a Cross Out list [below] will be part of the final draft.)

Cross out list for Terry. Known facts are blackened.

Case Studies

(Commentary for providing a remedial program for two students, of different ages and backgrounds, based on profiles provided by a Cross Out list [below] will be part of the final draft.)

Cross out list for Jim. Known facts are blackened.

Case Studies

(Commentary for providing a remedial program for two students, of different ages and backgrounds, based on profiles provided by a Cross Out list [below] will be part of the final draft.)

Pat's Cross Out List.

Case Studies

(Commentary for providing a remedial program for two students of different ages and backgrounds, based on profiles provided by a Cross Out list [below] will be part of the final draft.)

Skip's Cross Out List.

LOOK AHEAD
TEACH STRATEGY
CHOOSE AND USE
SPEED UP!
CHECK UP!

Evaluating Mastery of Basic Facts

(Questions teachers ask)

1. **When are facts "mastered"?**

Basic fact mastery involves giving *correct* answers *quickly* (normally within 2 seconds), and being *consistent* in this performance over time. Any assessment in the area of basic facts would consider these criteria.

2. **What do written timed tests really tell us about fact mastery?**

Unless individually monitored, these tests only tell us the number of facts a child can correctly answer within a given time. They don't tell us which facts are memorized vs which are calculated. They don't tell us *how* answers to unknown facts are derived. They do provide a good starting point by indicating which students are "pretty good" and which students need a lot of help in the area of basic facts.

3. **What is the role of longer timed tests (60-100 facts)?**

These tests are most useful for up front diagnostic assessment and for occasional general achievement testing of basic fact mastery. For example, one might give such a test about the second or third week of school to determine fact retention over the summer and assist with instructional planning. At the end of a semester this test might be readministered to determine long term retention or progress. These tests would not be given on a daily basis.

4. **How much time should be allowed for timed tests?**

The time allowed should be a function of children's normal writing speed. These speeds vary due to differences in age and motor coordination. An individual's writing speed can be determined by counting the number of digits a child normally can write in one minute. A child who can write fifty digits per minute should be able to write answers to thirty basic facts per minute. This time allocation takes into account 2-digit vs. 1-digit responses.

5. **What is the role of short "jiffy" quizzes in testing mastery of basic facts?**

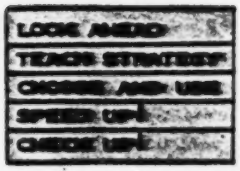
Evaluation is tied to teaching. Hence, a first level of assessment in the area of basic facts relates to children's mastery of each new group of facts studied (doubles, 9s, ...). When students have had the chance to investigate cues and hints for remembering a group of facts, and have had opportunity for practice, both oral and written quizzes could be used. JIFFY QUIZZES are good for this purpose. These quizzes are:

- Short--containing only 6 or 8 facts.
- Frequent--even daily over a period of several days.
- Limited at first only to the new facts being studied.

Cumulative review quizzes test retention over time. We recommend that JIFFY QUIZZES again be used for this purpose.

6. **What are alternatives for tracking progress during fact learning?**

It is important to keep students attuned to their progress and goals for improvement. Keeping Cross Out sheets and the teacher's record of these (p. 15) up to date is one way of doing this. In a regular classroom it will be manageable to do this only with 4 or 5 students, typically those most in need. An alternative is to have students keep a personal bar graph or record of the number of *correct answers* on basic fact quizzes. Teachers may find it useful to keep a master class list. Knowing exactly which facts a student knows will help the teacher emphasize these facts with individual children, the p. 15 charts might be useful for checking off facts as they are learned.



A Sound Basic Fact Program:

HOW TO INTEGRATE THESE IDEAS WITH THE SCHOOL ADOPTED MATH TEXT

1. **When teaching facts, group by STRATEGY FOR RECALL, not by size of related sum or product.** Look at each page from the child's perspective. Does the child have an efficient way of answering unknown facts? If the child's only recourse is to finger count, skip count, guess or use objects--teach relevant strategies before assigning text pages. (Turn to other topics in the meantime to give you time for this.)

2. **Plan ahead. Find and teach/review prerequisites to success** in fact learning. See the p. 16, Look Ahead SPONGE ACTIVITIES.

3. **Plan activities for developing strategies with students.** We can't just "tell" children retrieval strategies. We can provide experiences which will nurture correct, efficient thinking in fact learning.

4. **Do the "easy" addition and multiplication, then "easy" subtraction and division facts before turning to "other" remaining facts for each operation.** Typically "easy" facts for each operation are given below.
   - **Addition:** Count Ons (1, 2, or 3 being added); Doubles (4 + 4, 7 + 7...); Facts with 0; Easy 10-Frame facts (especially 5 + 4); and pattern 9s (9 + 6; 4 + 9...).
   - **Subtraction:** Doubles (12 - 6; 14 - 7...); Count Backs (-1,-2,-3 facts); Count Ups (8-6, 11-9, -- when "numbers are close." Most have 1, 2, 3, for answers); Facts with 0; Easy 10-Frame facts (10-6; 10-3;...); Pattern 9s--certainly by Gr. 2 (14-9; 16 - 9...) and Zero Finger-Facts -- beginning about semester 2, Grade 2 (all facts with teen minuends: 14 - 6; 16 - 7...).
   - **Multiplication & Division:** Picture 2s; clock 5s; pattern 9s; facts with 0 and 1. (Warning: Skip counting and rote chanting of the "times tables" will, in the long run, not provide retention and real mastery for average and slower students. Use records/tapes which present the "tables" in rote, sing song fashion only in final, consolidating stages of fact learning.)

5. **Initially delay subtraction/division.** Take pages out of sequence if necessary. Be sure students have firm mastery of related addition and multiplication facts before dealing with a group of subtraction and division) facts. This may mean doing a Geometry, Measurement or

other section of the book until this mastery is reached. With grade 1 level beginners, try to reach the subtraction concept work just before Christmas. With other learners you should be able to review concepts and facts for "easy" subtraction facts relatively fast--and get on to facts needing most attention.

6. **When you introduce/review the subtraction (division) concept and the"easy" facts for these operations:**
   a) Try to avoid emphasis too early on "families";
   b) Initially model the subtraction concept using the language and action of "take away." Use the language of "sharing" for division;
   c) Then, for each group of "easy" facts, teach a strategy for getting the answer;
   d) When students feel comfortable with a strategy, then **provide activities and exercises which require students to "add (multiply) to check".**
      Example 1: Have students "Break a Stick" (of unifix cubes)--apart and back together to illustrate the addition/subtraction relationship.
      Example 2: Have students use rows of small objects to illustrate the multiplication/ division relationship.
      Example 3: Have students write the addition and multiplication fact that "checks" beside selected subtraction and division facts they have completed.

7. **Use a Cross Out list** (near the end of the year with beginners; start near the beginning of the year with others). Do this for the lowest 4-5 students in your group. Allow use of this list for homework and tests. KEEP THE LIST UPDATED!--Additional facts should be learned and crossed out every week.

8. **Have students do only certain problems on a page (e.g. the Count Ups, the subtraction Doubles; the "Zero Finger" facts, the Clock 5s).** Allow students to skip others for which they have no efficient retrieval strategies. Reassign them later. On each page, have students write in facts which do not appear on the page but which "fit" a particular strategy you are studying. Younger students could refer to a wall chart or board list to carry out this task. Call facts by "name": Doubles, Count Ups,Ppattern 9s, and so on. Encourage students to do so.

   Provide activities which help students "sort out" groups of facts by strategy.

9. **Systematically check concept understanding for each operation.** On a regular basis, have students circle a problem on a page and use and counters or draw a picture to show the answer makes sense.

10. **Systematically carry out (short) daily review drills** on facts previously presented. Once a fact is introduced, it should be included in a drill at least once every week. This is necessary for any group in which there are students who lack fact mastery.
    Grades 1 -2: Plan for a 5-minute addition and a 5-minute subtraction drill daily (once facts for both operations have been introduced)--no matter what else is done during math period.
    Grades 3^{+}: Alternate days. Drill addition/subtraction one day, and multiplication/division the next--no matter what else is done during math period on those days.

11. **Integrate problem solving**, applications and extensions (e.g. graphing and mental math) with fact work.

`^^^^^^^^^^^^^^^^^^^^^^^^`

For unknown facts:

- **Sponge activities to build prerequisites.**
- **Activities to teach relevant strategies.**
- **Activities and follow up written exercises to practice each strategy and to sort out when each should be used.**
- **"Speed Up" activities and follow up written exercises (include "fun" pages)--systematic review practice on facts to insure long term retention.**

Using Blank Gamepaths for Basic Fact Practice

Need: Gamepath and chance cards, p. 234; cardboard spinner (see pattern below--child pokes pencil through center hole and spins like a top); game marker for each player; paper and pencil to tally points earned during play; deck of cards with facts needing practice. It is possible for each student to have a different deck of cards during play.

To Play (with 2-3 students):

1. Mix the chance cards and put them in a pile, face down on the gameboard. Then mix the fact cards and put them in a pile, face down. Put all game markers on Start.

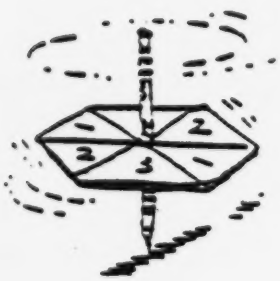
2. Each player draws a card and tells the answer. Player with the highest answer goes first.

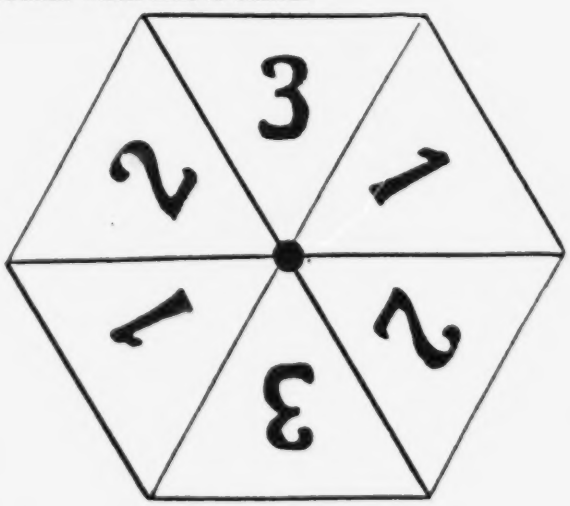
3. In turn, players draw a card and tell the answer. Use answer key or calculator to check when necessary.

   * If wrong, take a "pit stop." Move off the gamepath,just beside your space. Keep the card and study it. Give right answer and get back on the gamepath at your next turn.
   * If right, put card in discard pile. Then spin and move that many spaces.

4. First to reach Finish wins the round. Mix the cards and play again.

5. **Winner of game**: Player who has won most rounds when time is called.





Sample Letters to Parents

(To be provided in final draft.)

Game Directions for Parents

Suggestions for Use by Teachers: Cut out the directions and clip or staple them to a copy of the related game sheet/wheel for students to take home. You also may want to send home a list of facts to be practiced. **These should be selected according to the strategies currently being developed or practiced.** For example, the Number Fact Wheel should only be used for -2, -3, or -9 facts--which easily "fit" specific strategies.-- After the -4 through -8 facts are mastered, the wheel can be used for review. Similary, the wheel should only be used for +2, +5 and +9--and only later with other facts (after they are initially mastered). You may want to tell parents the strategy hints you provide if children do not know facts in the list you provide.

Try to Beat. (A "speed up" game for math fact practice.)

Use the attached sheet or simply number two columns 1 - 10 as shown. The name of the person playing with the child should be entered by the * at the top and bottom of the 2nd column.

Select 15 number facts (flash cards/facts written on paper). Study these with the child. Then one by one, show and read a fact aloud, and allow a short time (say, 2 seconds) per fact for the child to write the answer in the column headed "Me" on the sheet.

The child tries to write the correct answer before you give it. If the child can do this, the answer counts as 1 point in the column tally. If the child answers incorrectly or is unable to respond before you give the answer-- then the child should write the problem and its answer in the 2nd column, and whoever is playing with them gets 1 point in the column tally.

After 10 of the 15 facts have been called, add up the points in each column to see who wins the round. Play again! See who wins 2 out of 3 rounds. The game can be made more challenging by the child aiming to beat by as many times as possible or by increasing speed in answering.

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Suggestions for Use by Teachers: Cut out the directions and clip or staple them to a copy of the related game sheet/wheel for students to take home. You also may want to send home a list of facts to be practiced. **These should be selected according to the strategies currently being developed or practiced.** For example, the Number Fact Wheel should only be used for -2, -3, or -9 facts--which easily "fit" specific strategies.-- After the -4 through -8 facts are mastered, the wheel can be used for review. Similary, the wheel should only be used for +2, +5 and +9--and only later with other facts (after they are initially mastered). You may want to tell parents the strategy hints you provide if children do not know facts in the list you provide.

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Try to Beat

Game 1		Game 2	
1.		1.	
2.		2.	
3.		3.	
4.		4.	
5.		5.	
me ☐	* ☐	me ☐	* ☐

Game 3		Game 4	
1.		1.	
2.		2.	
3.		3.	
4.		4.	
5.		5.	
me ☐	* ☐	me ☐	* ☐

Doubles Concentration (A strategy-building card game for addition/subtraction doubles.)

Use ordinary Concentration rules, match related picture and fact--and tell the fact answer in order to "keep" the matching pair.

Seven is Heaven (A "speed up" game for math fact practice.)

Give each child a copy of the "Seven is Heaven" score sheet.. Enter seven number facts on your copy of the score sheet to form a calling sheet. Call out the facts one at a time and have the children write their answers on their sheets. The length of time allowed for students to write an answer should gradually be decreased from one game to the next:

Example: In the first game, allow up to 5 seconds for each answer; in the second allow 4 seconds for each fact; in third, 3 seconds; and in the fourth game, 2 seconds for each answer.

Use the same seven facts for each game, but call them in a different order each time. As children become more sure of themselves, you might cut the response time to 4, then 3, 2 and 1 second(s) per fact for consecutive games.

A child's score in this game is given by the least time for which a score of 6 or more correct is obtained. For example, if a child scores 6 out of 7 when the facts are presented at the rate of one every 5 seconds, a score of 50 is earned: the time taken to present the 7 facts. In another game, if the child scores 6 (or 7) out of 7 when the facts are presented at the rate of one every 4 seconds, the new score earned is 40. (Had the child only scored 5 out of 7 at the 4-second rate, then the previous score of 50 stands.)

Both speed and accuracy may be built up further by increasing the speed of presenting the facts and by raising the number required to "score" (e.g. from 6 to 7).

Seven is Heaven

Game 1

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____

TOTAL ☐

Game 2

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____

TOTAL ☐

Game 3

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____

TOTAL ☐

Game 4

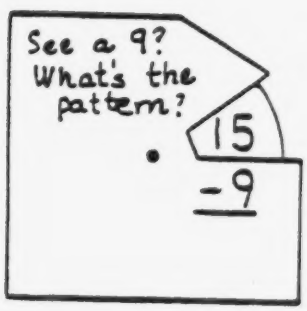
1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____

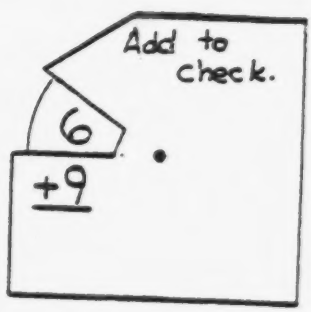
TOTAL ☐

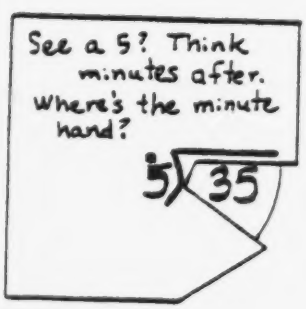
Number Fact Wheel

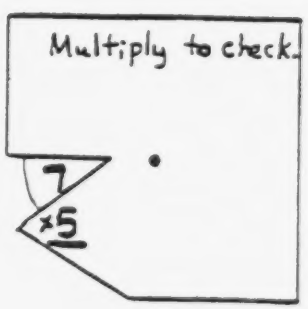
(A "speed up" game for math fact practice which also emphasizes the add/multiply to check idea.)

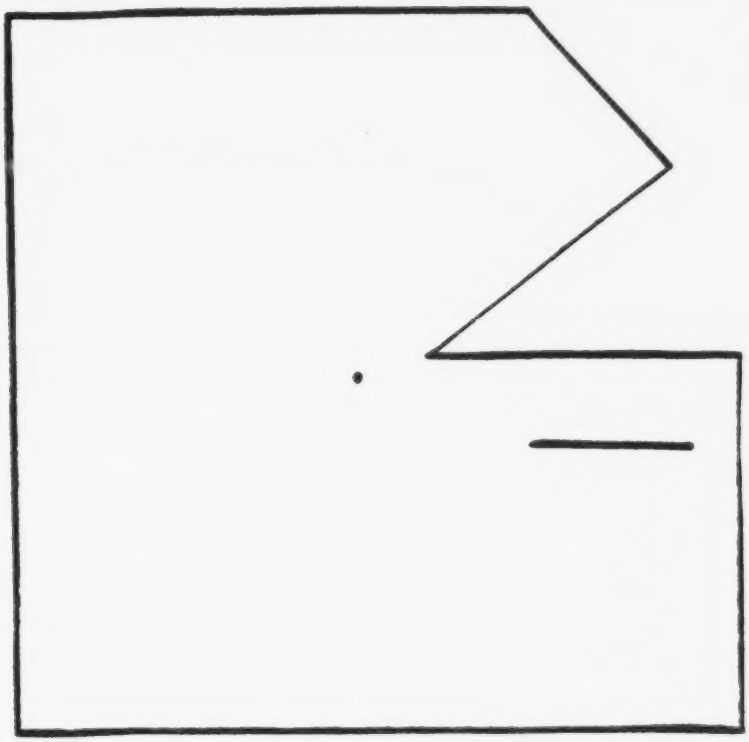
Students should first answer, one by one, each subtraction/division fact--then turn the wheel over and add/multiply to check. Students can keep track of the number correct; or of the time taken to get back to the "green dot" starting point--and try to better their record the next time they play.

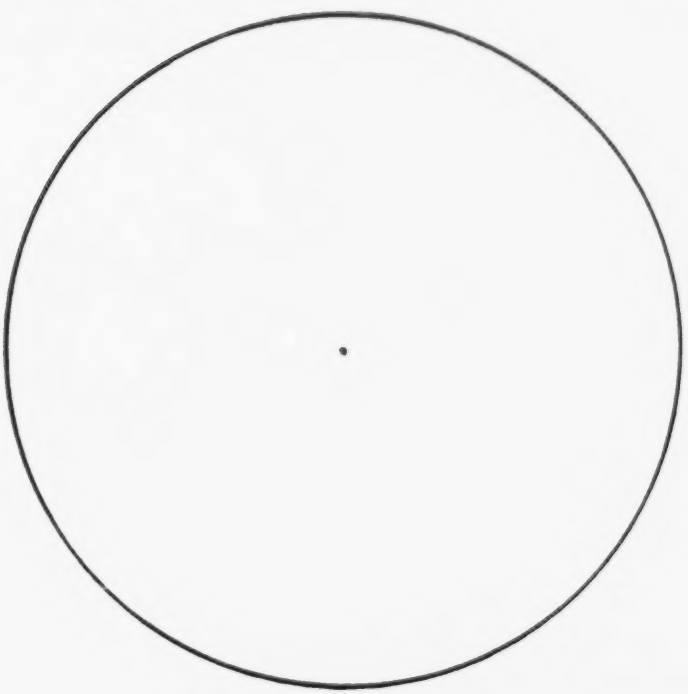












Extensions of the Basic Fact Program

- Concept Review
- Mental Math
- Problem Solving/Applications

(To be provided as part of the final draft)

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Glossary of Terms

1. **Count Backs** (count downs) are the -1, -2, -3 subtraction facts.
   Examples: 9 - **2** = 7; 8 - **3** = 5; 4 - **1** = 3.
   Verbal prompt: "If the number you take is 3 or less, count back."

   Note: Some children think of the 10-frame for all Easy 10-Frame facts, including 10-1, 10-2, 10-3. See #6 below.

2. **Count Ons** are addition facts in which you are adding 1, 2, or 3.
   Examples: 4 + **3** = 7; 5 + **1** = 6; **2** + 6 = 8.
   Verbal prompt: "For 1, 2, 3 in addition: Start big and count on."

3. **Count Ups** are subtraction facts where the "numbers are close."
   Examples: **9 - 7** = 2; **8 - 5** = 3 ; **7 - 6** = 1.
   The difference (answer) in each of the above is 1, 2, or 3.

   Note: Some children think of the 10-frame for all Easy 10-frame facts including 10-9, 10-8, 10-7. See #6 below.

4. **Doubles (addition/subtraction)** are facts in which both addends are the same.
   Examples: 4 + 4 = 8; 7 + 7 = 14 (addition doubles) 8 - 4 = 4; 14 - 7 = 7 (subtraction doubles).
   Verbal prompt: "Picture the double."

5. **Doubles + 1** are those facts in addition and subtraction that are "one more than" a double.
   Examples: 5 + 6 = 11; 7 + 8 = 15; 11 - 5 = 6; 15 - 7 = 8.
   Verbal prompt: "Think doubles to help."

6. **Easy 10-Frame facts** are those addition and related subtraction facts that can easily be modeled in the 10-frame.
   Examples: 6+4 and other 10 sums; 5+4 and 4+5 (all but one in the frame); 10 - 4 (or 10 minus any other 1-digit number); 9-5 and 9-4.
   Verbal prompt: "See 10- fact? Think of the 10-frame to help."

7. **Families of facts** are the related facts (addition-subtraction **or** multiplication-division) that have the same whole, same parts/factors.
   Examples: 5 + 4 = 9; 4 + 5 = 9; 9 - 4 = 5; 9 - 5 = 4.
   8 x 3 = 24; 3 x 8 = 24; 24 ÷ 3 = 8; 24 ÷ 8 = 3.
   Verbal prompt: "When you know one, you know all four."

8. **Make 10, Add Extra facts** are the five harder addition facts (7+4, 7+5; 8+4, 8+5, 8+6) that can be solved quickly by visualizing greater addend in the frame, filling it, then adding the extras.

9. **"New" multiplication/division facts** in multiplication and division refer to those facts not previously introduced using another strategy.

10. Pair is a term used to refer to related addition-subtraction or multiplication-division facts.

Examples: 9 - 7 = 2 and 7 + 2 = 9 are pairs.
("7+2=9" is the "add to check" fact for 9-7=2.)
Verbal prompt: "When you know addition, you know subtraction, too."
35 ÷ 5 = 7 and 5 x 7 = 35 are pairs.
(5 X 7 = 35 is the "multiply to check" fact for 35÷5=7.)
Verbal prompt: "When you know multiplciation, you know division, too." We encourage children to "pair up" (add/ multiply) to check subtraction/division answers.

11. Partner facts are two related subtraction/division facts with the same whole and the same parts/factors.

Examples: 9 - 2 = 7 and 9 - 7 = 2 are partner facts.
36 ÷ 9 = 4 and 36 ÷ 4 = 9 are partner facts.

First, one masters an "easy" group of subtraction or division facts. THEN, one can use these easier, known facts to help learn a whole new set of harder, unknown facts.

Referring to the first example above: the -2 count backs are relatively easy for many children. When the -2 count backs are well consolidated, some children can learn to use them to derive answers to related but unknown partner subtraction facts like 9-7.

Referring to the second example above: the pattern 9s are relatively easy for most children. When they are truly mastered, many children can learn to use +9s they know to derive answers to related but unknown partner ÷ facts like 36÷4.

Verbal prompts: "When you know one, you really know two!"
"Same whole, same **parts**/factors. They make good **part**ners."

12. 10 Sums are addition facts having a sum of 10.
Examples: 8 + 2 = 10; 6+4=10.
Verbal prompt: "Think: 'What 2 numbers fill the frame?'"

13. Turnarounds are addition/multiplication commutatives.
Examples: 8 + 7 = 15 and 7 + 8 = 15.
6 x 8 = 48 and 8 x 6 = 48.
Verbal prompt: "Turnarounds in addition (multiplication) give the same answer. When you know one fact, you really know two."

14. "Twice as Much" facts (multiplication) are those "harder" multiplication facts whose answers are twice those of easier, known facts.
Examples: **4 x 6** (2 x 6 = 12, so 4 x 6 is twice as much: 24.)
6 x 3 (3 x 3 = 9, so 6 x 3 is twice as much: 18.)
Verbal prompt: "An even factor? Do half, then double it."

15. **Zero facts** are important in all operations. The following list highlights how zero facts are treated in each operation.

Operation	Verbal Prompt	Examples
+ Zero Facts	"+ 0 makes no change."	4 + 0 = 4; 0 + 4 = 4.
- Zero Facts	"Take none" (4 - 0 = 4).	"Take all" (4 - 4 = 0).
x Zero Facts	"x 0, always 0."	4 x 0 = 0; 0 x 4 = 0.
+ Zero Facts	"Nothing to share, so you get nothing."	(0 + 8 = 0).

"+ 0 is NOT ALLOWED" (A No! No!)

16. **"Zero Finger" facts** in subtraction are those with teen minuends (e.g. 14-8, 13-7,14-6, 15-7).
    Verbal prompt: "Take from 10 *and* _____."

Useful Materials for Teaching Basic Facts

ADDITION

	TIME & AGAIN ACTIVITIES	COUNT ONS	ZERO FACTS	EASY 10-FRAME FACTS	DOUBLES	MAGIC 9S	DOUBLES +1	OTHER 10-FRAME FACTS	NOTES
1. Flash Cards		✓	✓	✓	✓		✓	✓	
2. Individual Chalkboards		✓			✓	✓	✓		
3. Unifix cubes or other small counters			✓			✓			
4. 10-frame(s)	✓			✓		✓			
5. Dice or spinners									
6. Small cards/ notepaper (sometimes numbered)		✓							
7. Tag Board		✓							
8. Paper sack			✓						
9. Calculator (overhead, if possible)						✓			

Useful Materials for Teaching Basic Facts

SUBTRACTION

	COUNT BACKS	ZEROS	DOUBLES	"10-" FACTS	COUNT UPS	OTHER TEEN MINUEND FACTS	NOTES
1. Flash Cards	✓	✓	✓	✓	✓	✓	
2. Individual Chalkboards		✓	✓	✓	✓	✓	
3. Unifix cubes or other small counters	✓						
4. 10-frame(s)				✓			
5. Dice or spinners							
6. Small cards/ notepaper (sometimes numbered)							
7. Demonstration Clock	✓						
8. Hide & Seek Cards		✓		✓			
9. Scrap paper Masking tape				✓			
10. Count Up Cards					✓		

Useful Materials for Teaching Basic Facts

MULTIPLICATION

	PICTURE 2S	'ONE A WEEK'	CLOCK 5S	PATTERN 9S	1S AND 0S	TWICE AS MUCH FACTS	SQUARES AND THE 'MINER'	NOTES
1. Flash Cards	✓	✓	✓	✓	✓	✓	✓	
2. Individual Chalkboards	✓					✓	✓	
3. Unifix cubes or other small counters								
4. 10-frame(s)								
5. Dice or spinners								
6. Small cards/ notepaper (sometimes numbered)		✓						
7. Clock and Chalkboard			✓					
8. Pages from an old telephone book	✓	✓						
9. Calendar pages with the first day as Sunday		✓						
10. Graph paper		✓				✓	✓	
11. "Gold Party" snack in gold foil							✓	

Useful Materials for Teaching Basic Facts

DIVISION

	PICTURE 2S	CLOCK 5S	PATTERN 9S	1S AND 0S	OTHER DIVISION FACTS	NOTES
1. Flash Cards	✓	✓	✓	✓		
2. Individual Chalkboards	✓	✓			✓	
3. Unifix cubes or other small counters					✓	
4. 10-frame(s)						
5. Dice or spinners						
6. Small cards/ notepaper (sometimes numbered)						
7. Clock		✓				
8. Construction paper strips: seasonal colors			✓			

Individual Chalkboards: Alternatives to Consider

(To be provided in final draft)

Using the Card Sets in the Basic Fact Program

(To be expanded in the final draft to cover all the card sets.)

Note: This approach to facts is recommended for all students except those few talented students who memorize with retention quickly and easily.

1. **Hide and Seek Cards**--can help students quickly master subtraction doubles because they use familiar pictures from addition. The cards also can reinforce/help develop children's ability to "add to check" subtraction answers.

   Introduce as below, but have students answer orally rather than form the written equation the first few times. Show the whole picture on the card. Have students write the number seen (e.g. **12** eggs).

   Hide all: "How many eggs did I hide?" (12). Open one flap: "How many eggs did I take out of hiding?" (Children write 12-**6**). "How many eggs are still hidden?" Children complete the equation: 12 - 6 = **6**)]. In subsequent lessons and activities, children can use the cards to get answers; write first then check written subtraction answers; write the addition fact that checks the subtraction answer...

2. **10-frame cards.** 10-frame "flash" activities should be related to work with numbers 0 - 10 and be reviewed with lst, 2nd, and other groups who have not yet mastered all 100 addition and all 100 subtraction facts. **Take the time to develop mental imagery in association with the 10 frame.** *Doing so can help beginners avoid finger counting for 4+6, 4+5, and all "Easy 10-Frame subtraction facts. The frame can also be used to help with 7 + 4; harder addition facts (e.g. 8 + (4,5,6); and 7+5); and all subtraction facts with teen minuends, starting near the end of grade 2.*

   First basic activity: "How many did you see in the frame?" Work especially for instant sight recognition of 4 to 9 dots in the frame. "When the first row is full--that's 5. We can count on from there to tell the number when 6 - 9 dots are in the frame. When the whole frame is full, that's 10 dots."

   To wait until children already are working with addition or subtraction facts like those listed in the previous paragraph is "too late." Children need to be able mentally to picture the frame if it

is to be useful in fact work, and the mental imagery takes time to develop.

A second type of activity is the following. Flash the cards 4 - 9: "How many dots do you see?" (Write the number.) "How many empty spaces?" (Write that number.) THEN SHOW THE CARD AGAIN so children can check. (Sample comment: "Yes, 4 and 6 -- that makes 10.") For all facts listed above, children should first use the 10-frame to get answers. The next step is to challenge students to write answers first, then use the frame afterwards to check that what is written is correct.

Sample activity for 4 + 5 and 4 + 6. Tape the 10-frame picture cards on the wall or place them along the chalk ledge. Show one of the fact cards (4 + 5 or 4 + 6) and ask individual students (perhaps as they leave the room) to say the answer and then match it to the 10-frame picture that checks. *Variation:* Use subtraction flash cards for "Easy 10-Frame subtraction " facts, 9-5 and 9-4. Show students one of the cards and have them say the answer and then match it to the picture to check.

For harder + facts listed above: Use the frame to illustrate the strategy: "Make 10, count extra." For subtraction facts with teen minuends: use the frame to illustrate the "Take from 10, count extra."

Note: For early number work and addition, fill the frame left-right, top row first. For subtraction, "take away" dots/chips in the reverse manner (right/left, starting with the bottom row).

Teach a strategy (cards can help). Practice the strategy. Then provide "speed up" drills.

3. **Doubles cards.** Use to develop mastery of addition and of subtraction doubles (and later of multiplication and division 2s).

Use real objects before using the cards with pictured objects. Carry out activities which, in a "hands on" manner, meaningfully relate each picture to a written doubles fact. (Unless you help children "relate" to the pictures, they will be useless. If possible, require 16 crayons in lower grades---or get a 16-crayon box yourself and make it "special"; allow students to play tic tac toe and be use they recognize they have 9 spaces for the X or O--18 in all; show students real egg

cartons and demonstrate the way a half dozen is split off (NOT in 2 rows of 6--the eggs would spill out!); get a calendar where the 1st of the month is on a Sunday--laminate it and allow students to circle and count (if necessary) the seven days in each week--14 in all; ...)

- Introduce a few pictures a day rather than all on the same day.
- With older learners, use pictures only as needed to help with harder doubles.
- Provide lots of "match" activities: picture to fact; and fact to picture--prior to drill for speed.
- Create a bulletin board display of doubles. Some teachers make theirs "within reach" of children: (1) show fact and picture; (2) hide picture under fact, so child can lift the fact for picture help as needed. Students write or match answer cards to facts.
- Have students circle all the "two week" facts on completed worksheets; have them find and do just the doubles on pages you indicate; have students write any doubles they do not see on a page...

4. **Doubles+1 cards.** Use especially to develop mastery for 5 + 6, 6 + 7, 7 + 8 (and turnarounds).
   a. Display (along chalk ledge, on bulletin board) the doubles facts and pictures. One at a time give students the **picture** for one of the D + 1 facts. "Tape it on top of (under) the "matching" double. (Example: Students will be led by the picture for 6 + 7 to "6 + 6.")
   b. Discuss the two pictures: how they are alike and how different (e.g. 1 extra egg in 6 + 7). Have students write the fact illustrated in the new picture (6 + 7). Emphasize that since 6 + 6 is 12, 6 + 7 will be 1 more (13)--students count on from 12, not from 6 or 7.
   c. Repeat **a**, but this time tape the fact and related picture card back to back and give it to students fact side up. (They can use the back side as a hint, if needed.)
   d. Repeat **c**, but this time remove the doubles picture from the display, so that students are required to match written facts, a near double to a double, and tell the answer for both.
   e. Practice the strategy: "Use doubles to help."
      - Circle and do the near doubles on certain worksheets/pages first. Write the double that helps by each.
      - In flash card drill, have students answer just those addition facts in which the numbers are one different" (e.g. 6 +7).
      - Carry out "match" type activites: Doubles + 1 to Doubles written facts.
      - A quick drill: (see below). "Double the number and add 1." Then have students (some at the chalkboard) write the near double to match.
      - Given a near doubles fact (e.g. 5 + 6), have students circle the same color, match to, place on top of, tell or otherwise relate the "new" D + 1 fact to an easier, known double.
   f. Follow with "speed up" drills and written "fun" exercises (e.g. codes, puzzles...)